CS 362, HW6

Prof. Jared Saia, University of New Mexico

Due: April 29th

1. Prove via induction that any tree with n nodes has exactly $n-1$ edges. Don't forget to include the Base Case (BC), Inductive Hypothesis (IH) and Inductive Step (IS).

2. Problem 23-4 "Alternative MST Algorithms"

3. Prove Claim 1 from Single Source Shortest Paths Lecture (Lecture 10)

4. Prove Claim 2 from Single Source Shortest Paths Lecture (Lecture 10)

5. Problem 24-3 Arbitrage

6. Saia Trucking is a very safety conscious (and algorithm loving) trucking company. Given a pair of cities, they always try to find the *safest* route between that pair. They are thus faced with the following problem.

   There is a directed graph $G = (V, E)$, where the vertices represent cities and the edges represent roads. Each edge has a value associated with it that gives the probability of safe transport on that edge i.e the probability that there will be no accident when driving across that edge. The probability of safe transport along any path in the graph is the *product* of the probabilities of safe transport on each edge in that path.

   The goal is to find a path from a given node s to a given node t that maximizes the probability of safe transport. Describe an efficient algorithm to solve this problem.

7. In the **RADIO-TOWERS** problem, you are given (1) a set S of towers; (2) a set T of subsets of towers; and (3) a set of k radio frequencies. You must determine: Is it possible to assign exactly one of k possible radio frequencies to each tower in such a way that each tower in each

set in T has a unique radio frequency?

As an example, let $S = \{a, b, c, d\}$, $T = \{\{a, b, c\}, \{a, d\}, \{a, b, d\}\}$, and $k = 3$. Then the answer is YES since we can assign tower a frequency 1, tower b frequency 2, and towers c and d frequency 3, thereby ensuring that each tower in each set in T has a unique frequency. Prove that this problem is NP-Hard by reduction from a problem we have covered in class.