1. Prove Claim 1 about GenericSSSP in the SSSP lecture.

2. Prove Claim 2 about GenericSSSP in the SSSP lecture. Let \( l \) be the number of edges in the path. Your goal is to show that: “For all \( \ell \geq 0 \), if any path \( s \sim v \) has \( \ell \) edges then \( \text{dist}(v) \leq w(s \sim v) \) when the algorithm halts”. Do this by induction on \( \ell \). Show the base case (i.e. the goal holds when \( l = 0 \)). Write down the inductive hypothesis (i.e. for all \( j < l, \ldots \)) and then show the inductive step.

3. Saia Trucking is a very safety conscious (and algorithm loving) trucking company. Given a pair of cities, they always try to find the safest route between that pair. They are thus faced with the following problem.

There is a directed graph \( G = (V, E) \), where the vertices represent cities and the edges represent roads. Each edge has a value associated with it that gives the probability of safe transport on that edge i.e the probability that there will be no accident when driving across that edge. The probability of safe transport along any path in the graph is the product of the probabilities of safe transport on each edge in that path.

The goal is to find a path from a given node \( s \) to a given node \( t \) that maximizes the probability of safe transport. Describe an efficient algorithm to solve this problem.