1. Prove that any tree with $n$ nodes has $n - 1$ edges. Hint: to get a smaller subproblem, remove a leaf node, i.e. a degree 1 node, from the tree. To prove that any tree has a leaf node, note that trees do not have cycles and consider a path that starts at the root node.

2. A certain product is sold in bags of sizes 3, 5 and 8 kilos. Prove that given an unlimited supply of bags of these sizes, you can fill an order for exactly $n$ kilos for $n \geq 8$. Hint: Be careful about the base cases, and also the lower bound for your III.

3. Jesse wants to know the minimum number of bags needed to fill an order of $n \geq 8$ kilos assuming unlimited supply of bags of sizes 3, 5 and 8 kilos. Let $f(n)$ be the minimum number of bags needed.
   
   (a) Write a recurrence relation for $f(n)$. Hint: Use the minimum function in your recurrence relation; it may also be useful to use the value $\infty$ for some of your base cases.
   
   (b) Use your recurrence relation to fill in an array of $f(n)$ values for $n \in [0, 16]$.
   
   (c) Describe an algorithm to compute $f(n)$ for any given $n$ using your recurrence relation. What is the runtime of your algorithm? How might you augment your algorithm to return the actual set of bags used to achieve the minimum value $f(n)$? Hint: you can put back-arrows in your array.
4. Consider the following function:

```c
int f (int n){
    if (n==0) return 2;
    else if (n==1) return 5;
    else{
        int val = 2*f (n-1);
        val = val - f (n-2);
        return val;}
}
```

(a) Write a recurrence relation for the value returned by \( f \). Solve the recurrence exactly. (Don’t forget to check it)

(b) Write a recurrence relation for the running time of \( f \). Get a tight asymptotic bound (i.e. \( \Theta \)) on the solution to this recurrence.