CS 506, HW3

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Due: April 20th

You are encouraged to work on the homework in groups of about 2 or 3. You may turn in one writeup per group, but please certify that all members worked on each problem.

1. *This time please do this with the "fictitious play" algorithm discussed in class. Verify that your algorithm now converges to a Nash equilibrium!* Recall the game of Rock, Paper Scissors. Assume that: 1) when scissors and paper are played, the scissors player wins 1 and the paper players loses 1; 2) when rock and scissors are played, the rock the winner wins 1 and the loser loses 1; 3) when rock and paper are played, the paper player wins .1 and the rock player loses .1;[1] and a draw results in both players getting 0. Suppose you make two copies of the MWU algorithm to play this game with each other over many iterations. Both copies start with a uniform random probability (i.e. play each of rock/paper/scissors with probability 1/3), and then learn from experience based on the MWU rule. Presumably, the players' probabilities will eventually converge to some sort of equilibrium. Predict, based on thought, what the probabilities will be in the equilibrium. Now, run this experiment on Matlab or any other programming environment, and report what you've discovered.

   Now, consider the same game, with the two players still using MWU, but assume that the second player has an η value much smaller than the first. For example, player 1 has $\eta = .1$, and player 2 has $\eta = .001$. Will the players still converge to an equilibrium? Will it take the second player 100 times as long to converge as the first? Make predictions, and then run an experiment on your favorite programming environment. Report what you've discovered and explain.

[1] Really, how badly can a piece of paper hurt a rock anyway?

2. Show that the Johnson-Lindenstrauss projection approximately preserves dot products. In particular, let P be the random projection matrix, let x and y be two unit vectors in the high dimensional space, and then bound the probability that $|x \cdot y - Px \cdot Py| \leq \epsilon$. Hint: Consider $1/4(|P(x+y)|^2 - |P(x-y)|^2)$, and then use what we proved in class about preservation of the norms of difference vectors.

3. Give an example (i.e., a set of n points in $\mathcal{R}^n$) that shows that the Johnson-Lindenstrauss projection we discussed in class does not preserve L_1 distances to within even a factor of 2.

4. (From Sanjeev Arora's hw) Suppose we have two sets P and N of unit vectors in R^m with the guarantee that there exists a hyperplane $a \cdot x = 0$ such that every point in P is on one side and every point in N is on the other. Furthermore, the L_2 distance of each point in P and N to this hyperplane is at least ϵ. Show using the Johnson Lindenstrauss lemma that for any random linear mapping to $O(\log n/\epsilon^2)$ dimensions, the points are still separable by a hyperplane with margin $\epsilon/2$.

5. (Arora) Implement the portfolio management appearing in the notes for Lecture 11 in any programming environment and check its performance on S&P stock data (download from http://ocobook.cs.princeton.edu/links.htm). Include your code as well as the final performance (i.e., the percentage gain achieved by your strategy).

6. (adapted from Arora) There is a grayscale photo at: www.cs.princeton.edu/courses/archive/fall15/cos521/image.jpg. Interpret it as an n by m matrix and run SVD on it. What is the value of k such that a rank k approximation gives a reasonable approximation (visually) to the image? What value of k gives an approximation that looks high quality to your eyes? Attach both pictures and your code. (In matlab you need mat2gray function.)
   Challenge: Apply the JL projection to project the m columns to $\mathcal{R}^k$, where k is as chosen above, then project the columns back into $\mathcal{R}^n$ using the inverse projection (this operation will of course be lossy). Compare this J-L compressed image with the image obtained via SVD compression. How do they compare?

7. Write a 3-5 page update on your progress on the class project. Also prepare slides for a 15 minute in-class presentation.