CS 506, HW3

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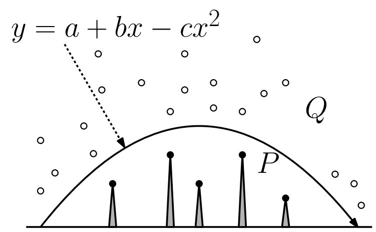

Figure 1. LP

1. (Adapted from Mount F'16) There is a set of n building tops, represented by points $P = \{p_1, \ldots p_n\}$ and m floating balloons, represented by points $Q = \{q_1, \ldots q_m\}$ (Figure 1). You have a cannon in $\mathbb{R}^2$ that has three controls labeled "a", "b", and "c". A projectile shot from this cannon travels along the arc $y = a + bx - cx^2$. Can you adjust the cannon so that the projectile travels above the set P, but below the set Q? You should determine this in $O(n + m)$ time. You can assume anything about the initial location of the canon so long as you clearly state it. Hint: Use Seidel's LP algorithm.
   Next, adapt your algorithm to find, in linear time in $n + m$, if there is a x-degree polynomial that separates any set of points P and Q in $\mathbb{R}^d$ for d and x both $O(1)$. FYI: This has applications to learning polynomial classifiers that separate P and Q.

2. You are given a set of points P in the plane. You want to find the smallest circle that contains all points. Give an efficient algorithm to do this. Hint: This can be done using an incremental algorithm and backwards analysis, as used in Seidel's LP algorithm.

3. Prove that the polar transformation discussed in class is incidence preserving and inclusion reversing.

4. Show that the Johnson-Lindenstrauss projection approximately preserves dot products. In particular, let P be the random projection matrix, let x and y be two unit vectors in the high dimensional space, and then bound the probability that $|x \cdot y - Px \cdot Py| \leq \epsilon$. Hint: Consider $1/4(|P(x+y)|^2 - |P(x-y)|^2)$, and then use what we proved in class about preservation of the norms of difference vectors.
   Next, give a counterexample that shows that the Johnson-Lindenstrauss projection does not preserve L_1 distances to within even a factor of 2. Your counter-example should be a specific set of n points in $\mathcal{R}^n$,

5. *During the Italian Renaissance, mathematicians challenged each other to public mathematical duels, wherein each duelist posed multiple problems to their opponent. These duels were high-stakes: the winner received fame and a lavish dinner bankrolled by their opponent for each of their unsolved problem.*[1] *The code of conduct required that all questions a duelist posed were previously solved by that duelist.*
   For this problem, you will pose your best Computational Geometry dueling question. In particular, write a question whose solution (1) uses tools learned in class since the last hw; and (2) does so in a creative or interesting manner that may not be readily apparent to your unfortunate dueling opponent. This problem will be graded both on how interesting your question is, and also on the correctness of your solution.

[1] For example, in 1535, Niccolo Tartaglia won 30 dinners at a Venetian tavern from his opponent, Antonio Fior, in a duel that involved many math problems related to solutions to the cubic equation.