

# CS 561, HW5

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*Due: October 30th*

1. (Probability) Solve Problem 5 on the midterm from 2013 at <http://www.cs.unm.edu/~saia/classes/561-f13/mid.pdf>
2. Problem 17-2 (Making Binary Search Dynamic)
3. Problem 22-4 (Reachability) <sup>1</sup>
4. Professor Curly conjectures that if we do union by rank, *without path compression*, the amortized cost of all operations is  $o(\log n)$ . Prove him wrong by showing that if we do union by rank without path compression, there can be  $m$  MAKESET, UNION and FINDSET operations,  $n$  of which are MAKESET operations, where the total cost of all operations is  $\theta(m \log n)$ .
5. Assume you are given a connected graph  $G$ . Give an algorithm that returns a vertex  $v$  in  $G$ , such that if  $v$  is removed,  $G$  is still connected. Motivation:  $G$  might represent a social network at a company and you want to choose some unlucky person to fire whose removal will not disconnect the company network.
6. Professor Moe conjectures that for any graph  $G$ , the set of edges  $\{(u,v) : \text{there exists a cut } (S, V-S) \text{ such that } (u,v) \text{ is a light edge crossing } (S, V-S)\}$  always forms a minimum spanning tree. Given a simple example of a connected graph that proves him wrong.
7. Exercise 23.1-2 (“Professor Sabatier conjectures”)
8. Exercise 23.1-3 (“Show that if an edge  $(u,v)$  is contained in some minimum spanning tree”)

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<sup>1</sup>The answer to this problem can be used in an efficient randomized algorithm for estimating the \*number\* of vertices that are reachable - we may see this later in this class.

9. Exercise 22.2-6 (“There are two types of professional wrestlers”)