CS 561, HW5

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Due: November 1st

1. (Probability) Solve Problem 5 on the midterm from 2013 at http://www.cs.unm.edu/∼saia/classes/561-f13/mid.pdf
2. Problem 17-2 (Making Binary Search Dynamic)
3. Problem 22-4 (Reachability) [1]
4. Professor Curly conjectures that if we do union by rank, *without path compression*, the amortized cost of all operations is $o(\log n)$. Prove him wrong by showing that if we do union by rank without path compression, there can be m MAKESET, UNION and FINDSET operations, n of which are MAKESET operations, where the total cost of all operations is $\theta(m \log n)$.
5. Assume you are given a connected graph G. Give an algorithm that returns a vertex v in G, such that if v is removed, G is still connected. Motivation: G might represent a social network at a company and you want to choose some unlucky person to fire whose removal will not disconnect the company network.
6. Professor Moe conjectures that for any graph G, the set of edges {(u,v) : there exists a cut (S,V-S) such that (u,v) is a light edge crossing (S, V-S)} always forms a minimum spanning tree. Given a simple example of a connected graph that proves him wrong.
7. Exercise 23.1-2 ("Professor Sabatier conjectures")
8. Exercise 23.1-3 ("Show that if an edge (u,v) is contained in some minimum spanning tree")

[1] The answer to this problem can be used in an efficient randomized algorithm for estimating the *number* of vertices that are reachable - we may see this later in this class.

9. Exercise 22.2-7 (“There are two types of professional wrestlers”)