

CS 561, HW5

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Due: November 1st

1. Solve Problem 2 on the midterm
2. Solve Problem 5 on the midterm
3. Problem 17-2 (Making Binary Search Dynamic)
4. Problem 22-4 (Reachability) ¹
5. Professor Curly conjectures that if we do union by rank, *without path compression*, the amortized cost of all operations is $o(\log n)$. Prove him wrong by showing that if we do union by rank without path compression, there can be m MAKESET, UNION and FINDSET operations, n of which are MAKESET operations, where the total cost of all operations is $\theta(m \log n)$.
6. Assume you are given a connected graph G . Give an algorithm that returns a vertex v in G , such that if v is removed, G is still connected. Motivation: G might represent a social network at a company and you want to choose some unlucky person to fire whose removal will not disconnect the company network.
7. Professor Moe conjectures that for any graph G , the set of edges $\{(u,v) : \text{there exists a cut } (S, V-S) \text{ such that } (u,v) \text{ is a light edge crossing } (S, V-S)\}$ always forms a minimum spanning tree. Given a simple example of a connected graph that proves him wrong.
8. Exercise 23.1-2 (“Professor Sabatier conjectures”)
9. Exercise 23.1-3 (“Show that if an edge (u,v) is contained in some minimum spanning tree”)

¹The answer to this problem can be used in an efficient randomized algorithm for estimating the *number* of vertices that are reachable.