CS 561, HW5

Prof. Jared Saia, University of New Mexico

Due: November 5th

1. You are given a chocolate bar of length n and width 1 that is made up of n chunks of width and length 1. Each chunk $1 \leq i \leq n$ has some positive value v_i.

   You must break the bar into exactly k pieces to share with your friends, where each piece consists of some number of contiguous, unbroken chunks. Your (greedy) friends then each get to chose their pieces, and they leave you with the piece with minimum value. Thus, your goal is to do the breaking in such a way that you maximize the value of the minimum value piece. The value of a piece is simply the sum of the value of the chunks in that piece.

   Give a dynamic program to solve this problem. What is the runtime of your algorithm? Note: This is a recent Amazon interview question.

2. Problem 17-2 (Making Binary Search Dynamic)

3. Professor Curly conjectures that if we do union by rank, *without path compression*, the amortized cost of all operations is $o(\log n)$. Prove him wrong by showing that if we do union by rank without path compression, there can be m MAKESET, UNION and FINDSET operations, n of which are MAKESET operations, where the total cost of all operations is $\theta(m \log n)$.

4. Problem 22-4 (Reachability) [1]

5. Assume you are given a connected graph G. Give an algorithm that returns a vertex v in G, such that if v is removed, G is still connected. Motivation: G might represent a social network at a company and you want to choose some unlucky person to fire whose removal will not disconnect the company network.

[1] The answer to this problem can be used in an efficient randomized algorithm for estimating the *number* of vertices that are reachable.

6. Professor Moe conjectures that for any connected graph G, the set of edges $\{$(u,v) : there exists a cut (S,V-S) such that (u,v) is a light edge crossing (S, V-S)$\}$ always forms a minimum spanning tree. Given a simple example of a connected graph that proves him wrong.

7. Exercise 23.1-2 ("Professor Sabatier conjectures")

8. Exercise 23.1-3 ("Show that if an edge (u,v) is contained in some minimum spanning tree")