ANTS on a Plane

Jared Saia

Joint with Abhinav Aggarwal

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ANTS problem

N agents start at node (*nest)* on infinite grid

Target is placed on node at distance **L**

Goal: Find the target ASAP

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Synchronous agents; no communication
Advice: bits to encode #agents, roles, etc

[Feinerman and Korman, 2017]

ANTS Results

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Target is placed on node at distance **L**

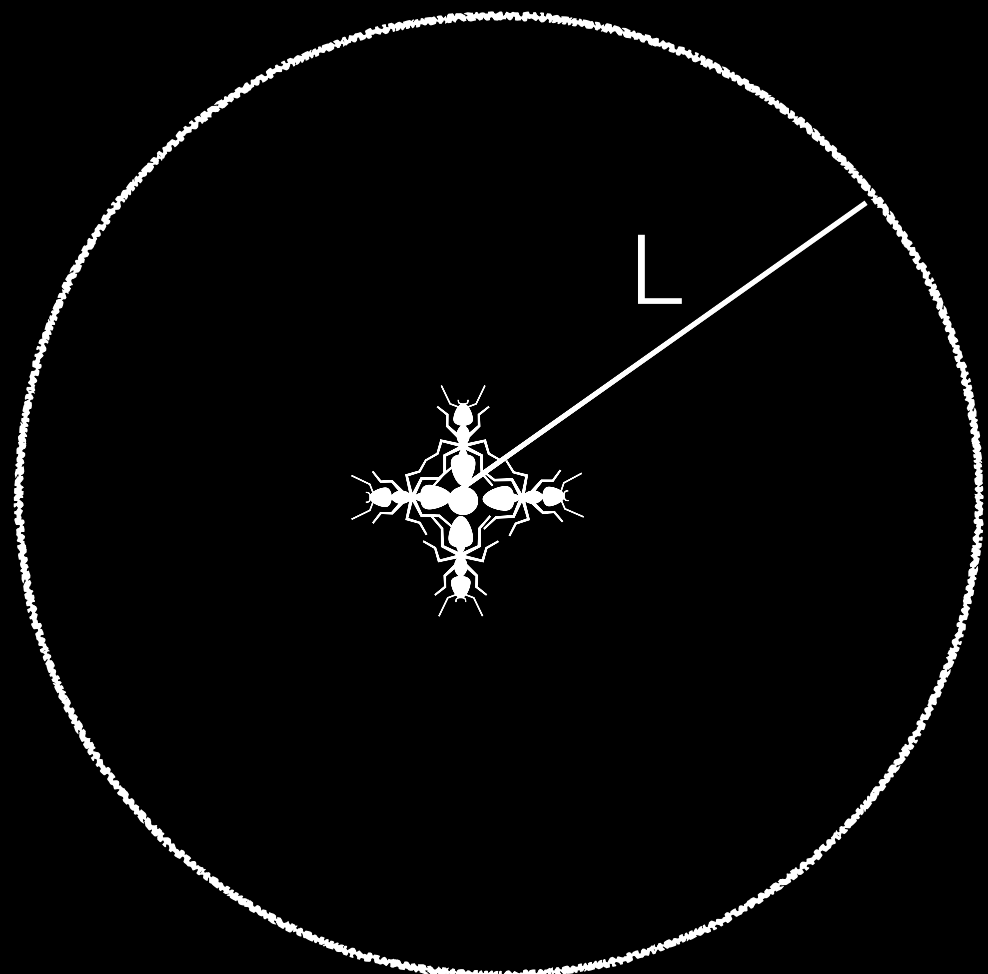
Advice: bits to encode #agents, roles, etc

$O(L + L^2/N)$ time with $O(\log \log N)$ bits advice

$O((L + L^2/N)\log^{1+\epsilon} N)$ time with no advice, for any fixed $\epsilon > 0$

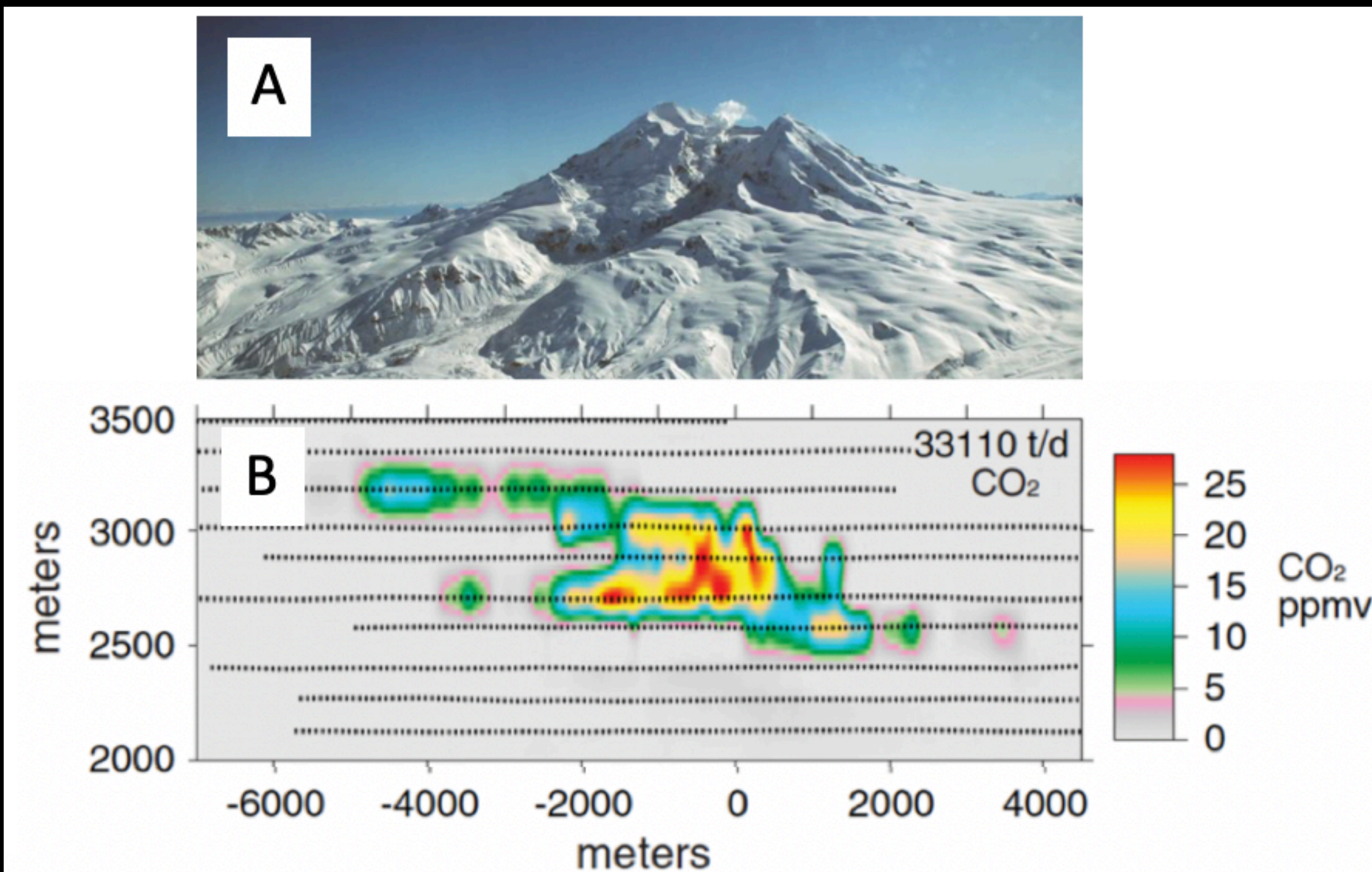
[Feinerman and Korman, 2017]

$O(L + L^2/N)$ is Optimal



Area to Search $\approx L^2$
N agents
Need $\approx L^2/N$ to search area
Plus L time to reach target

Motivation: Drones seek CO_2



Melanie Moses

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Problems:

- Choosing grid granularity
  - Too low: May miss target
  - Too high: Computational load on agents
- Hard to handle different target shapes

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Solution: Formulate problem on Euclidean plane

Target Shape

What target shape can we handle?

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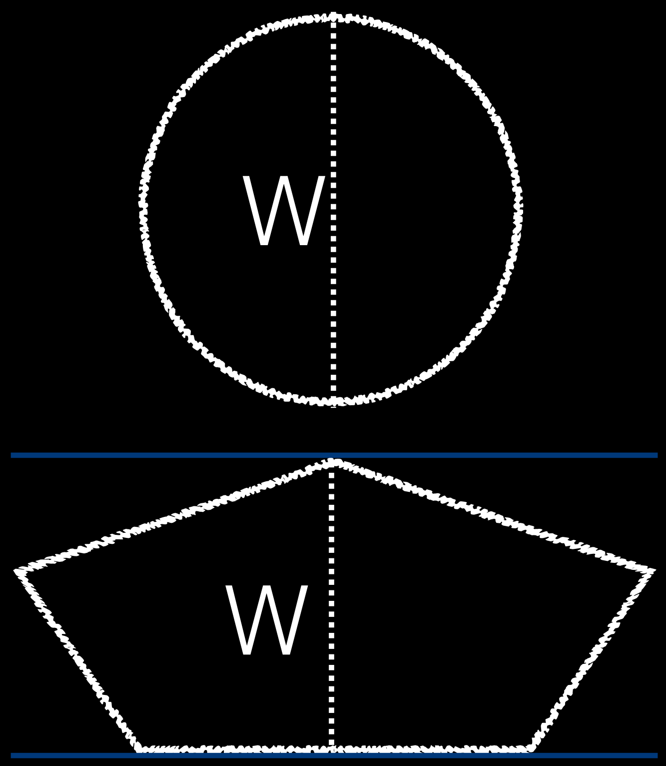
A convex shape

Width: Smallest distance between two parallel lines touching boundary but not interior

L is distance to segment W. Assume $W \leq L$.



Our Result - No Advice

$$O\left(\left(L + \left(\frac{L^2}{NW}\right)\right)\log L\right)$$ search time

[F&K '15] $O((L + L^2/N)\log^{1+\epsilon} N)$ time, for any $\epsilon > 0$

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If $t < N$ agents removed by adversary

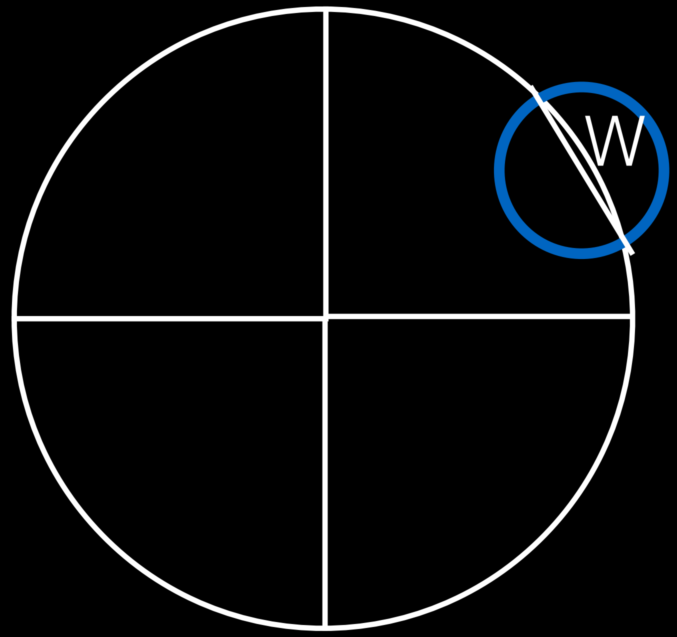
$$O\left(\left(L+\frac{L^2(t+1)}{NW}\right)\log L\right)$$ search time

Spokes

Spoke: line segment from nest and back

Say target is on circle of circumference 1

How many spokes are needed to find it?



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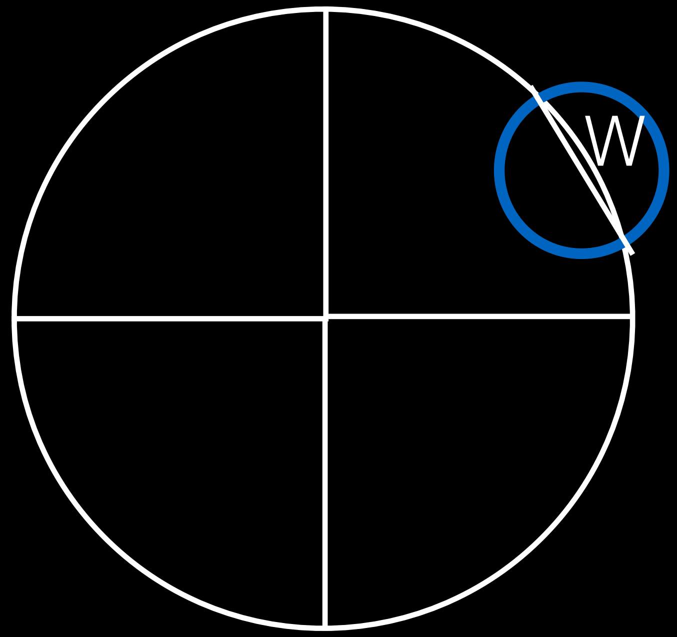

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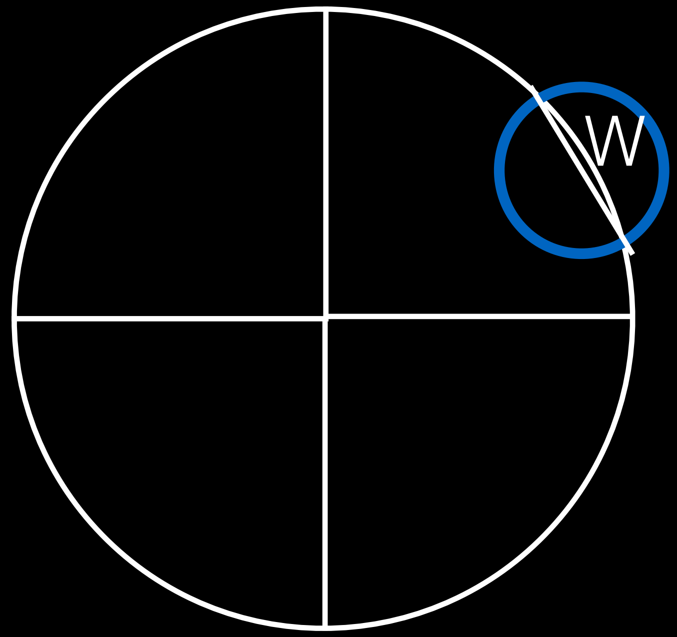
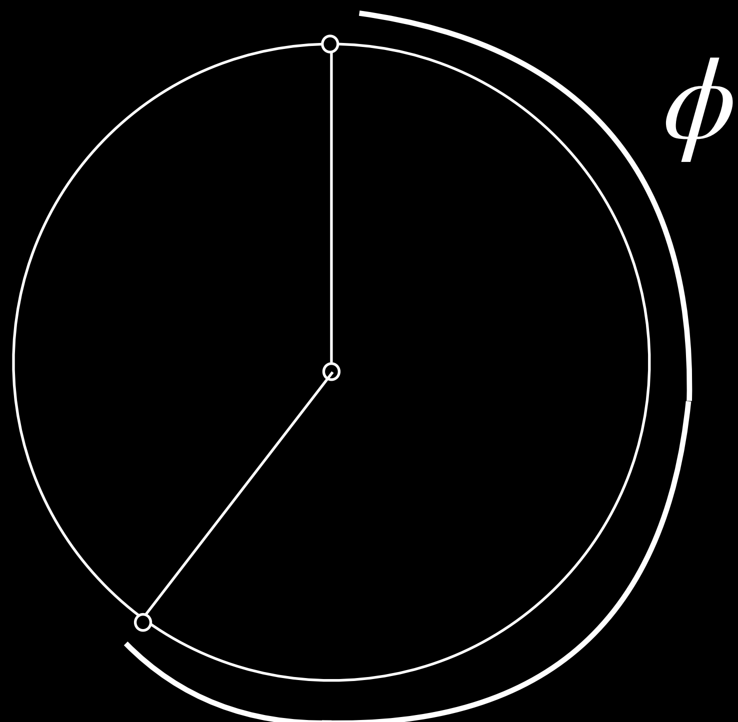


Suffices: 1/W evenly spaced spokes

How to get “evenly” space spokes, when don’t know W in advance?

Idea: Use ϕ



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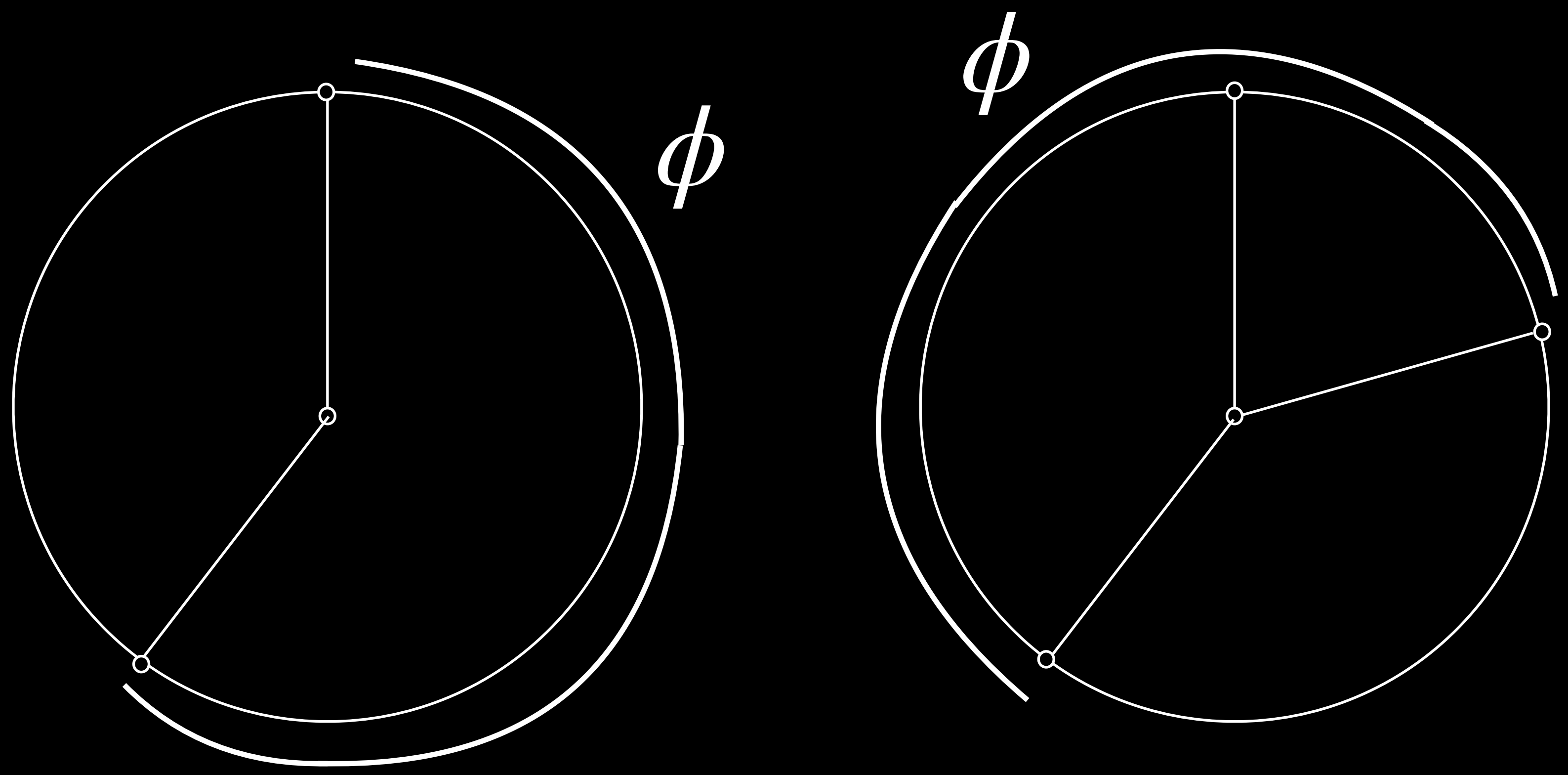

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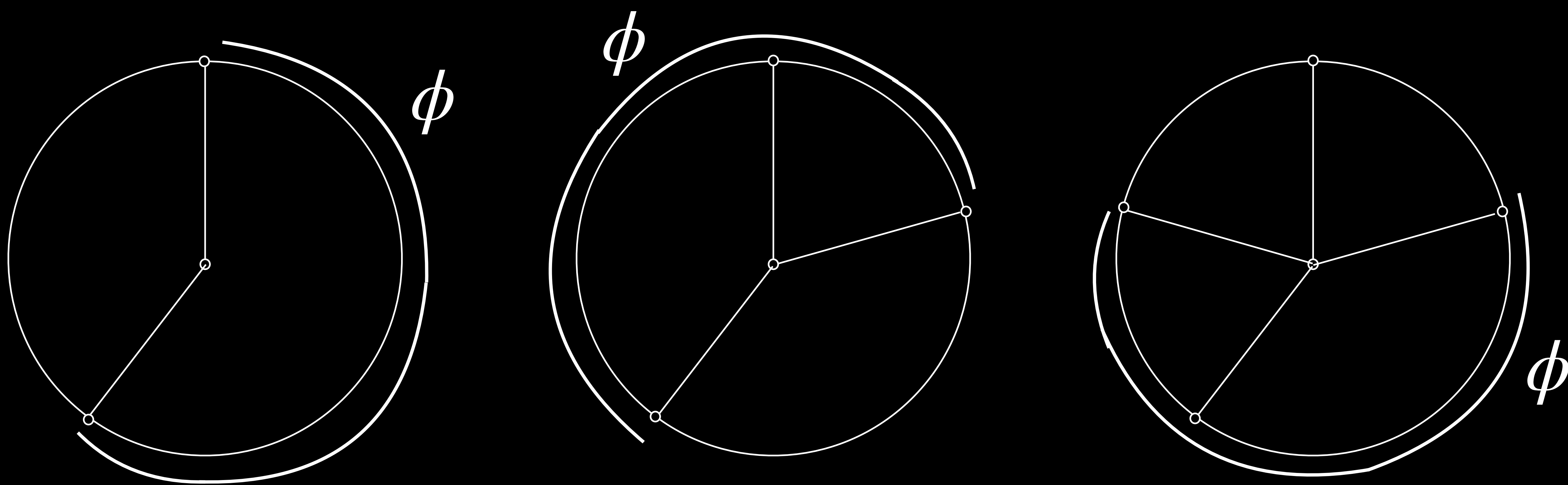

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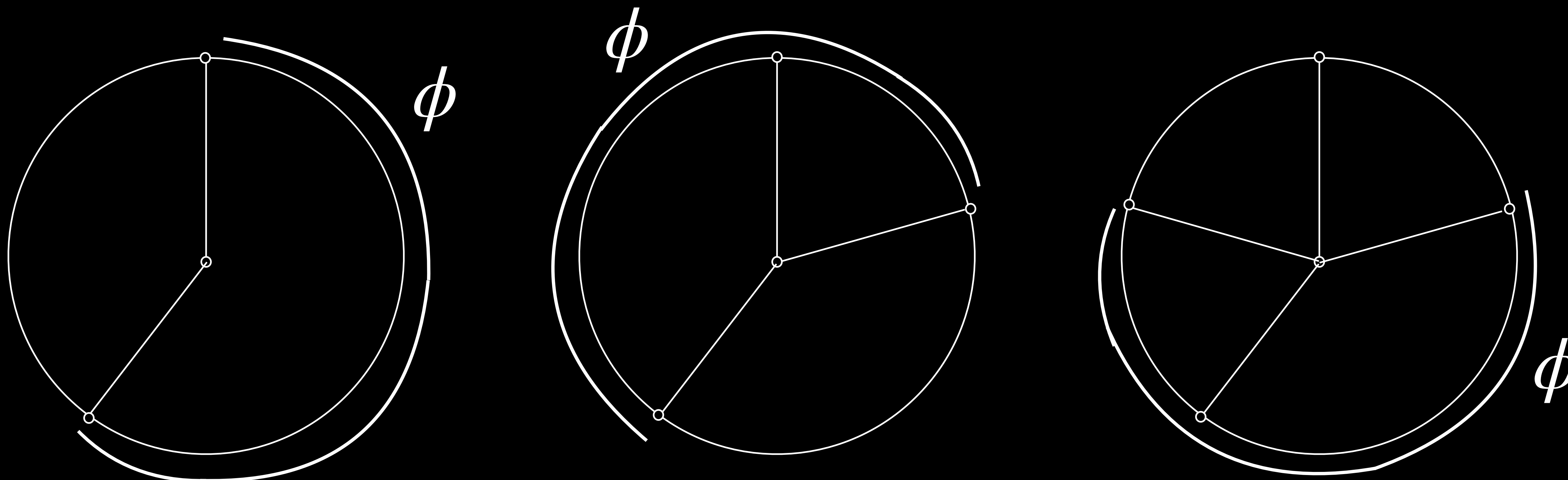

[Swierczkowski, ’58] M points placed at arc distance ϕ → arc length between any neighboring points is O(1/M)

Moreover, ϕ minimizes hidden constant over all numbers

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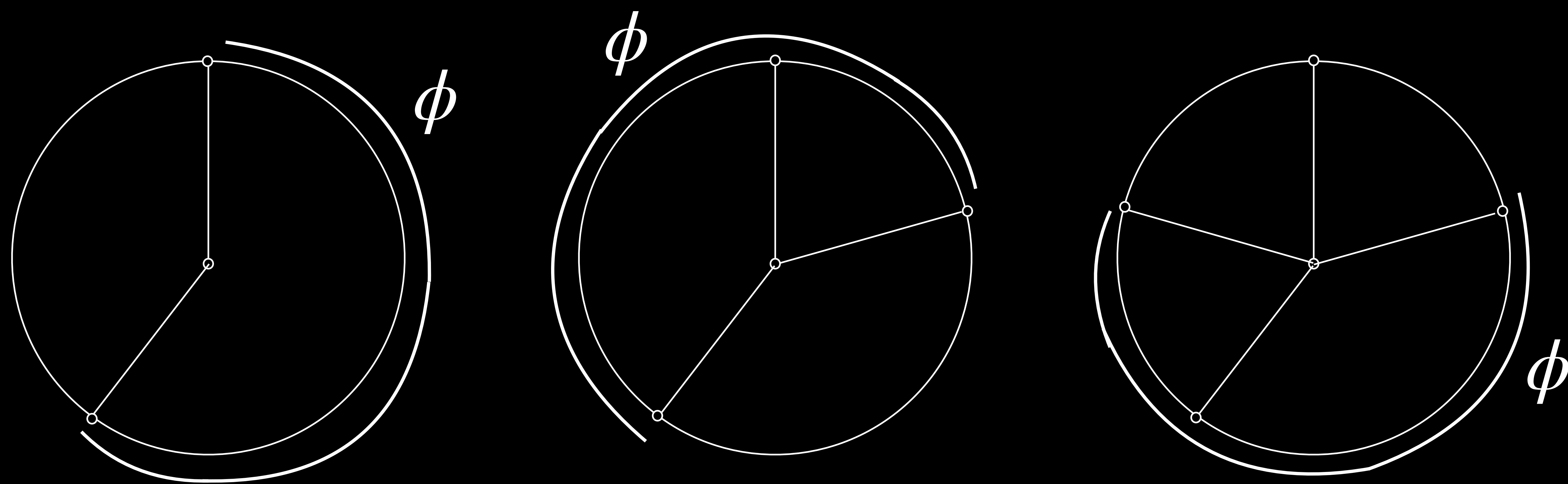

arc lengths between neighboring spokes are $\approx 1/4$

[Swierczkowski, ’58] M points placed at arc distance $\phi \rightarrow$ arc length between any neighboring points is O(1/M)

Moreover, ϕ minimizes hidden constant over all numbers

Why does $\phi \to$ well-spread?

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To get this, set $y = 1 + \frac{1}{y}$. Solving yields: $y = \phi$

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"very irrational" →well-spread

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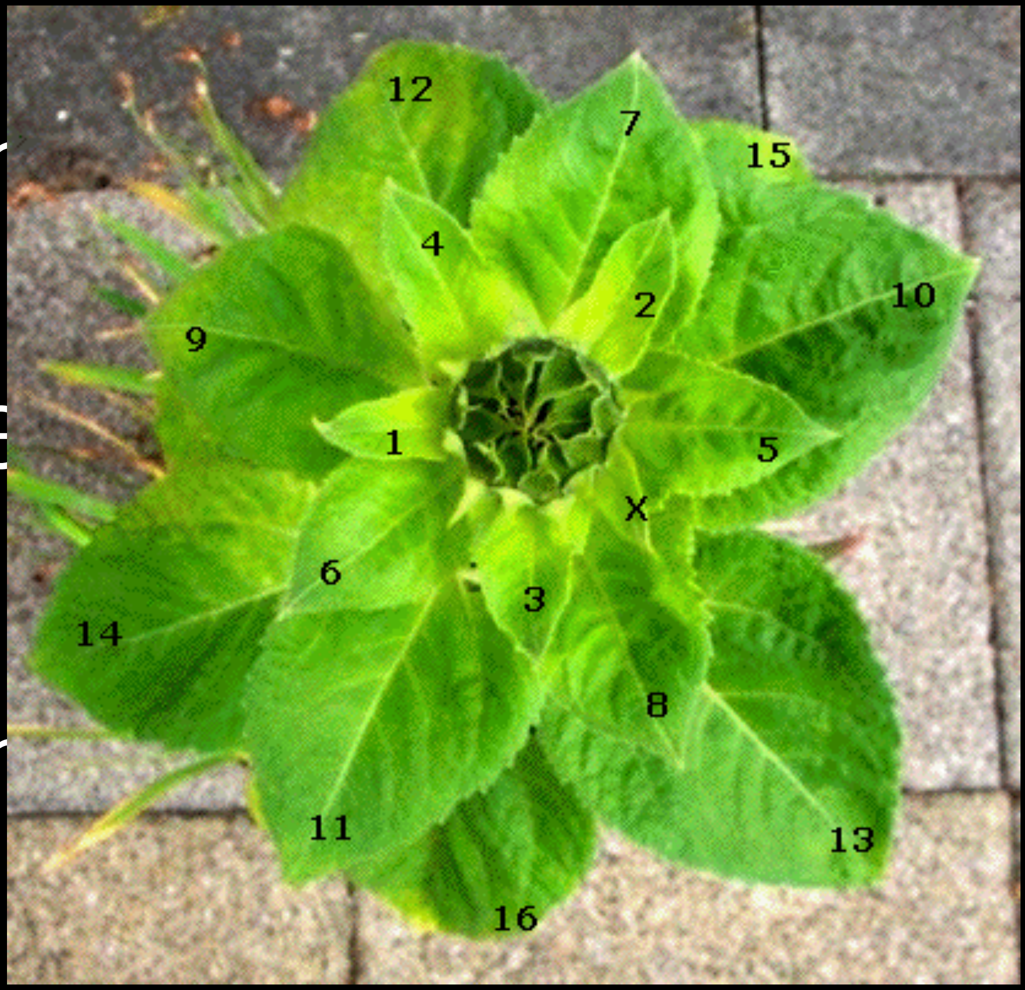
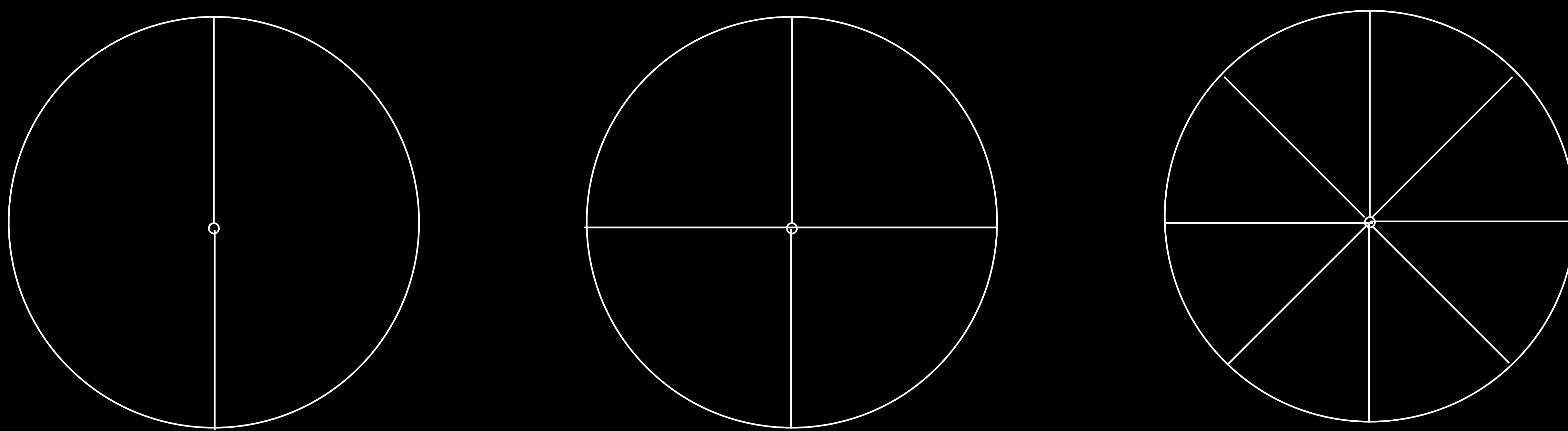


Why not powers of 2?



Q: Why not have 2^i evenly space spokes in iteration i?

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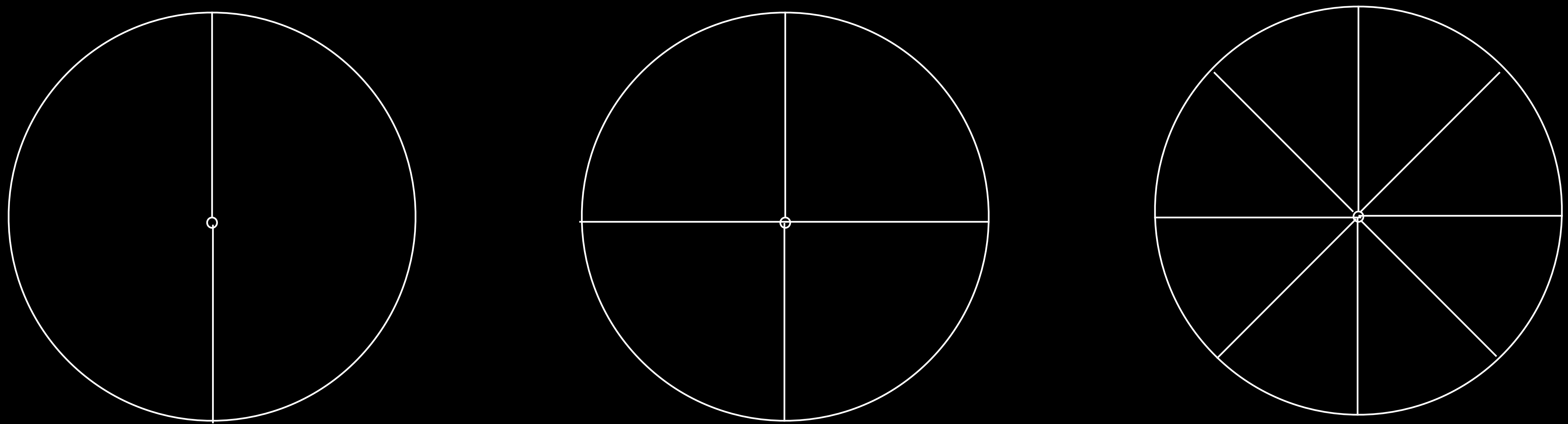
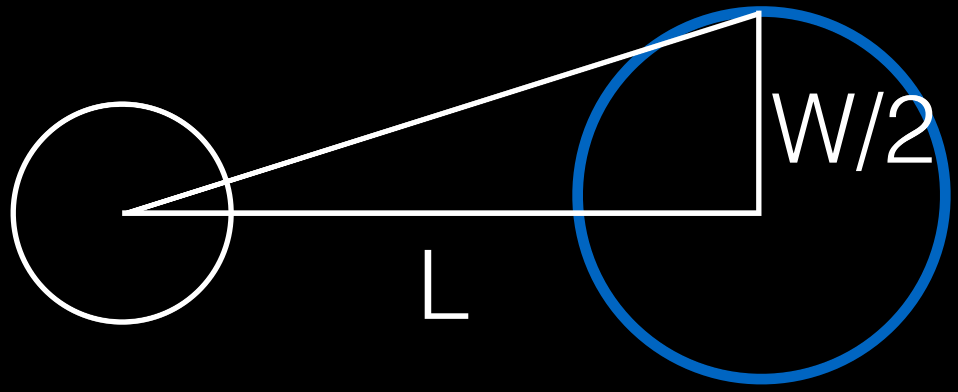


Q: Why not have 2^i evenly space spokes in iteration i?

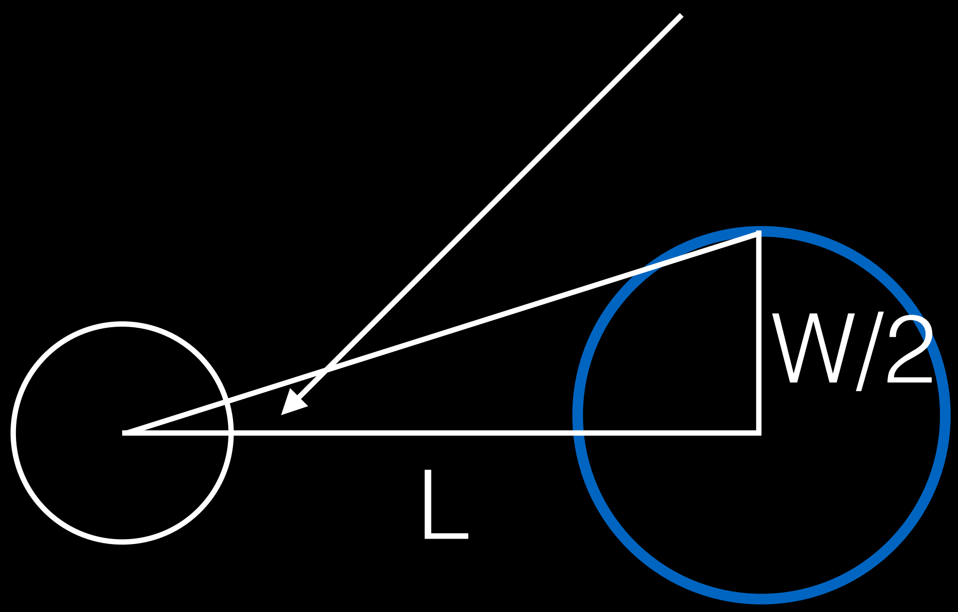
A1: Off from optimal number of spokes by $\leq$ factor of 2
A2: Requires memory for the counter, and also adds algorithmic complexity.

How many spokes?



How many spokes?

$$\theta = \sin^{-1} \frac{W}{2L}$$



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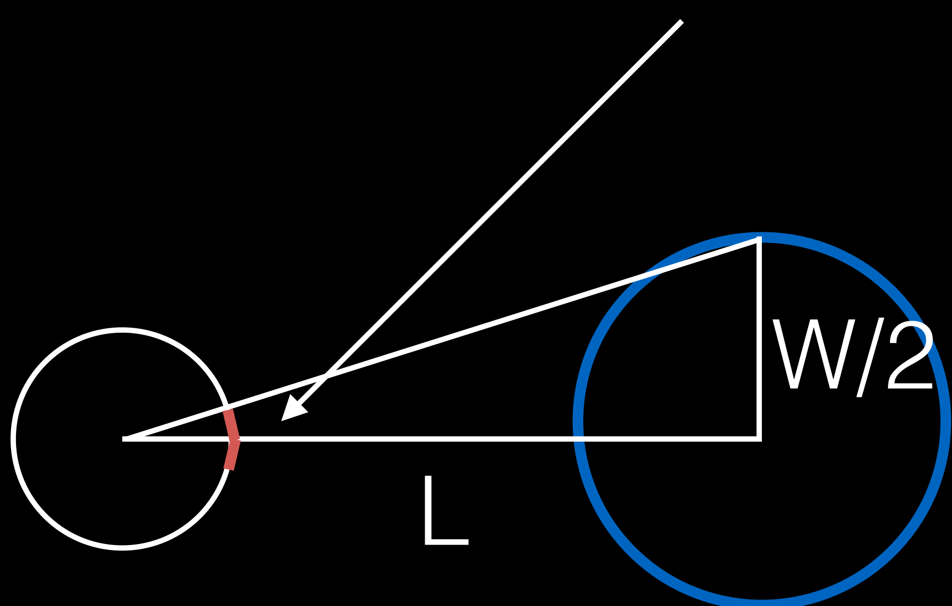

Let α be arc length on unit circle

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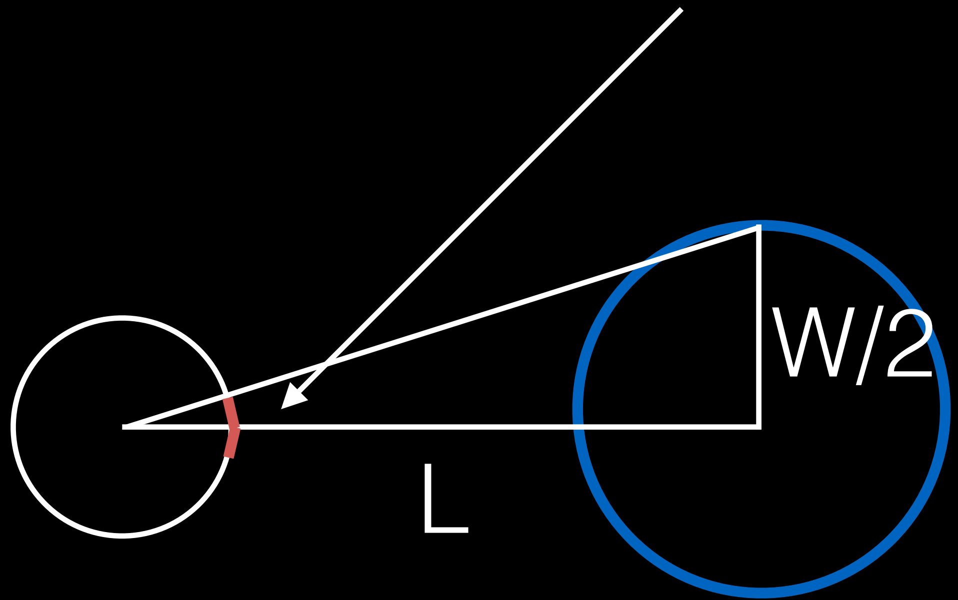
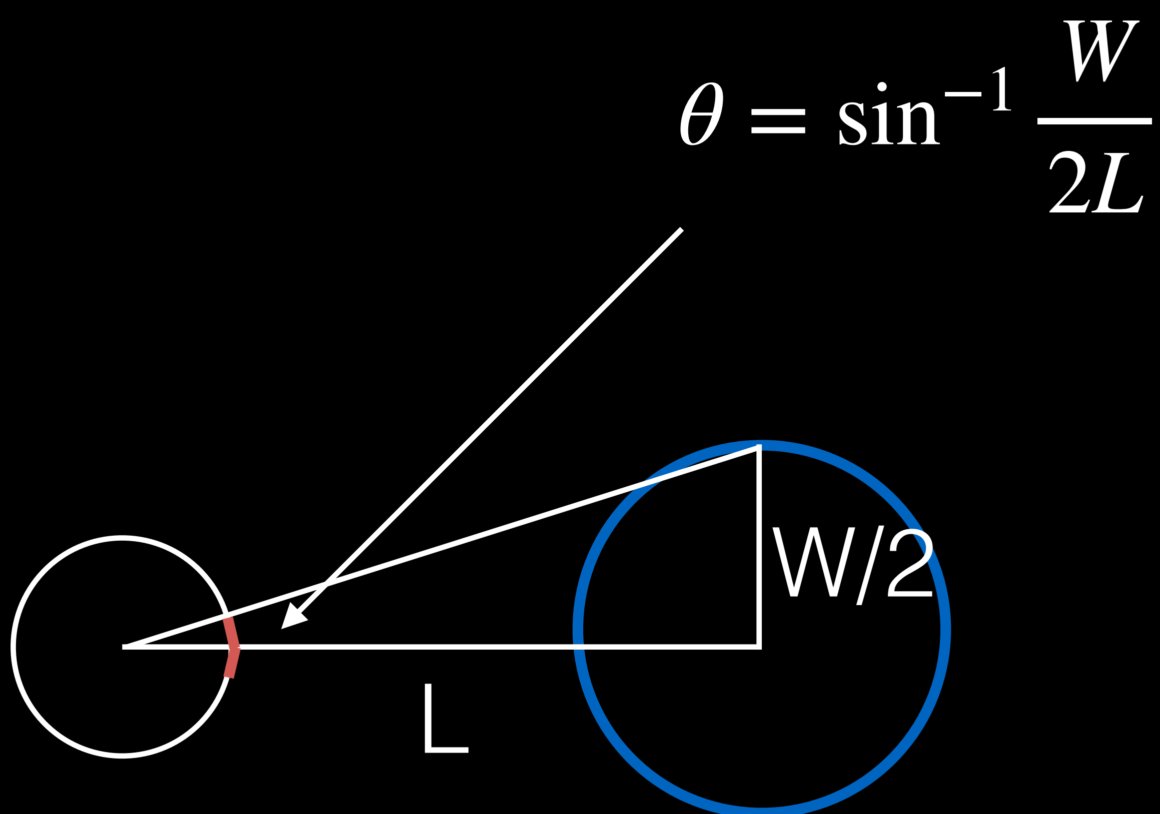


Let α be arc length on unit circle

$$\alpha = \frac{1}{\pi}\sin^{-1}\frac{W}{2L}$$

Power Series: $\sin^{-1} x \geq x$, for $|x| \leq 1$

How many spokes?



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So by [Swierczkowski, '58], need O(L/W) spokes

How many spokes?

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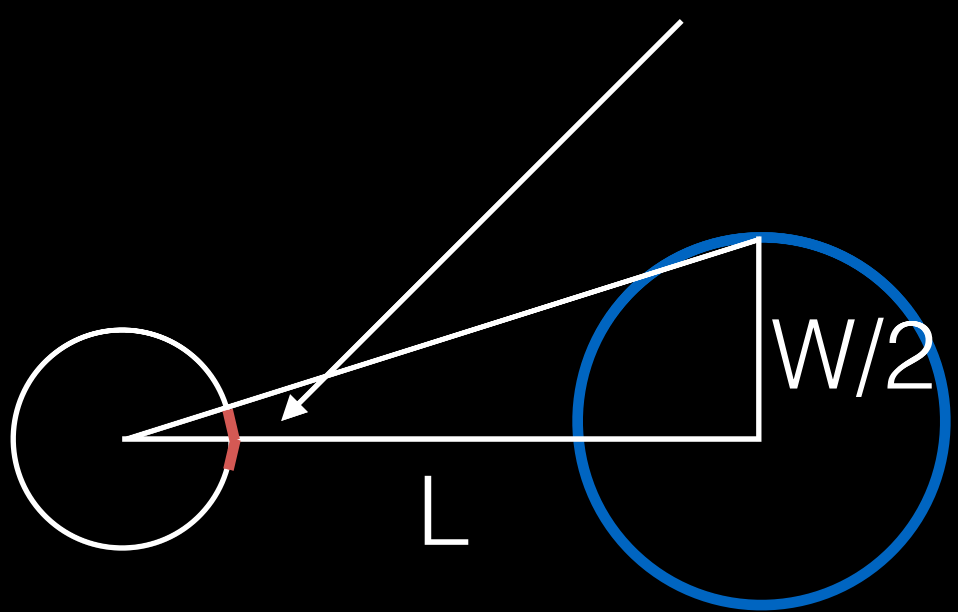

$O(L^2/W)$ search time!
If know L in advance

Let α be arc length on unit circle

$$\alpha = \frac{1}{\pi} \sin^{-1} \frac{W}{2L}$$

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How to handle unknown L?

Spoke Length

spokes

	1	2	4	8	16
1					
2					
4					
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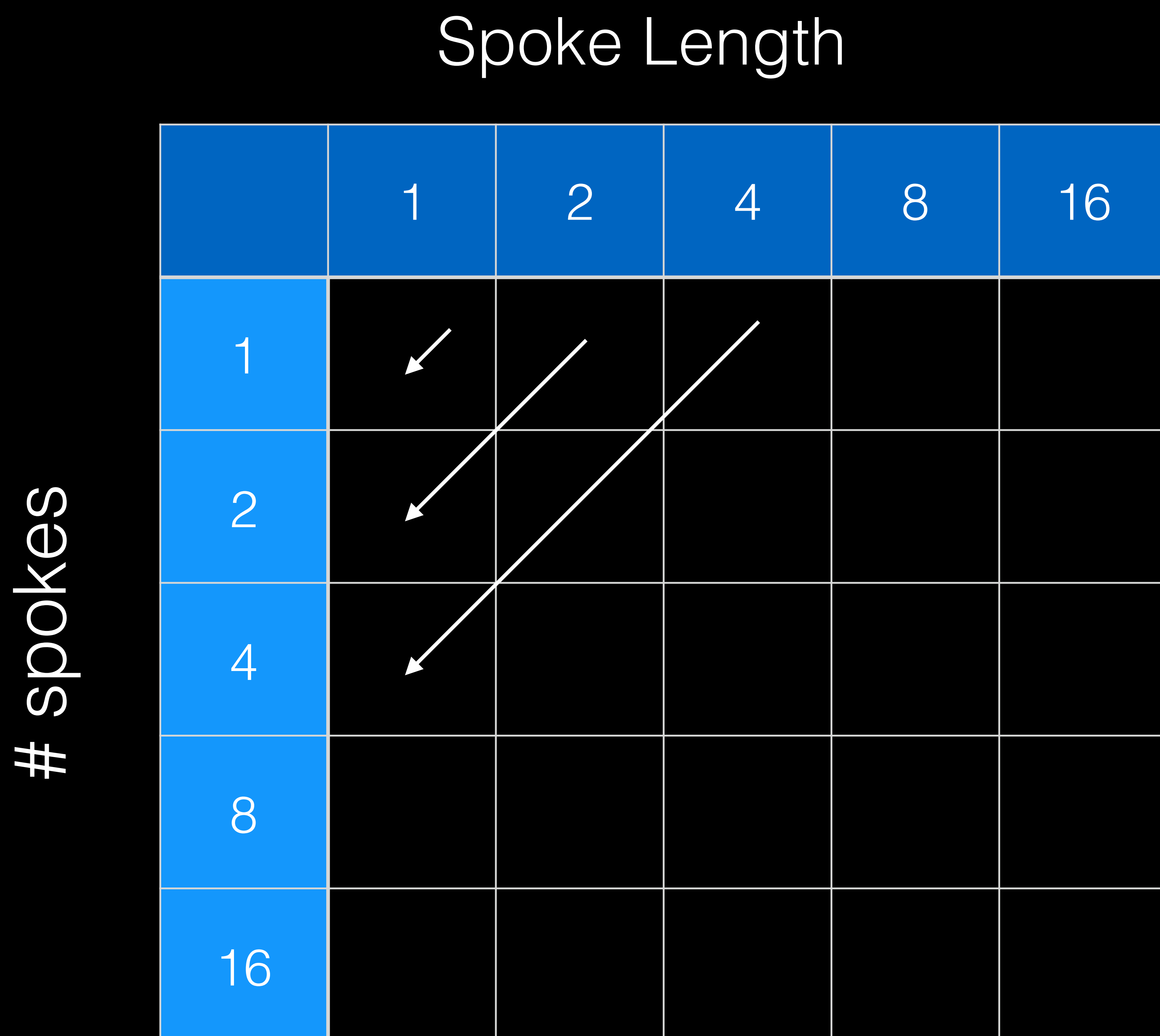

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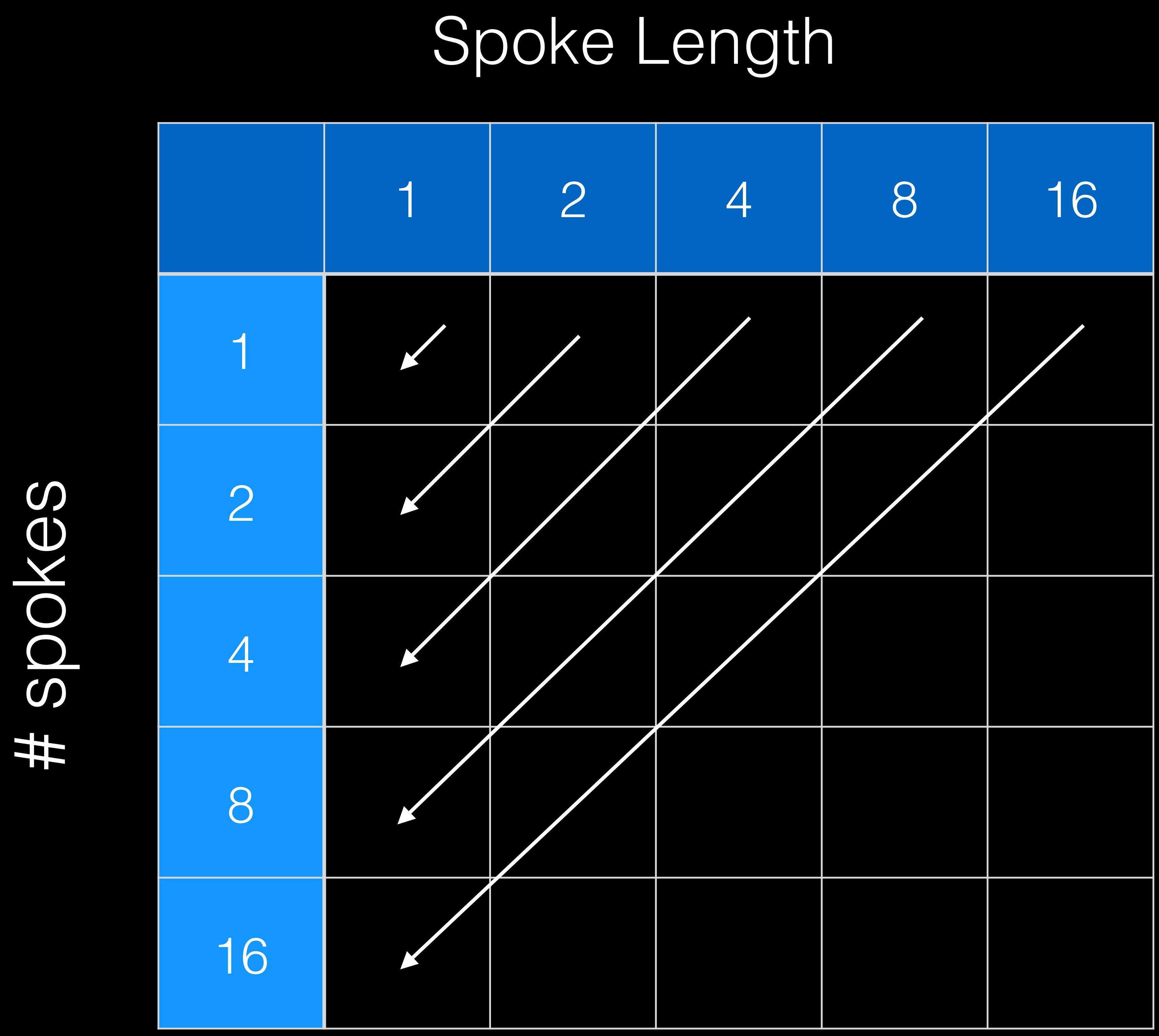

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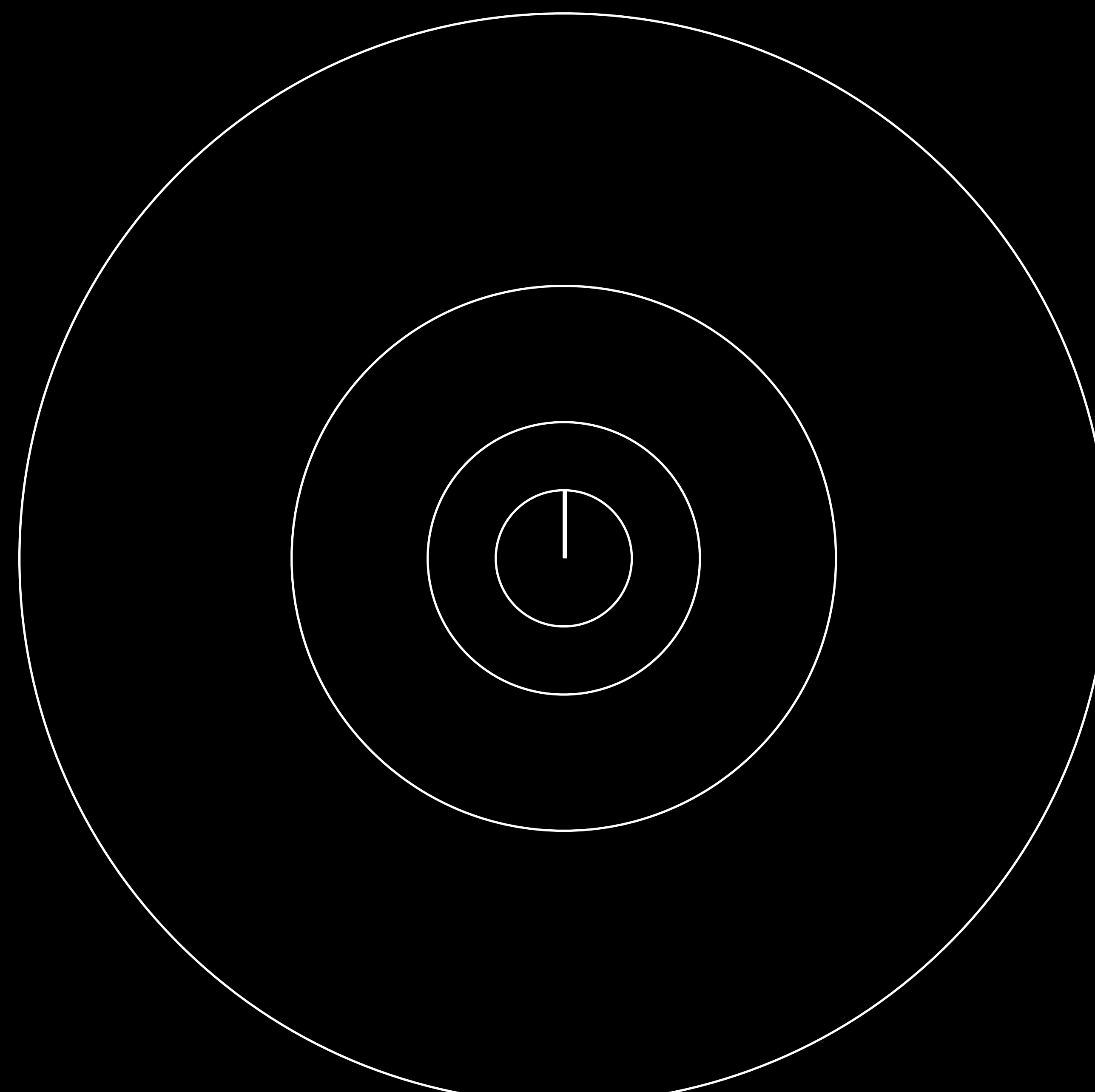

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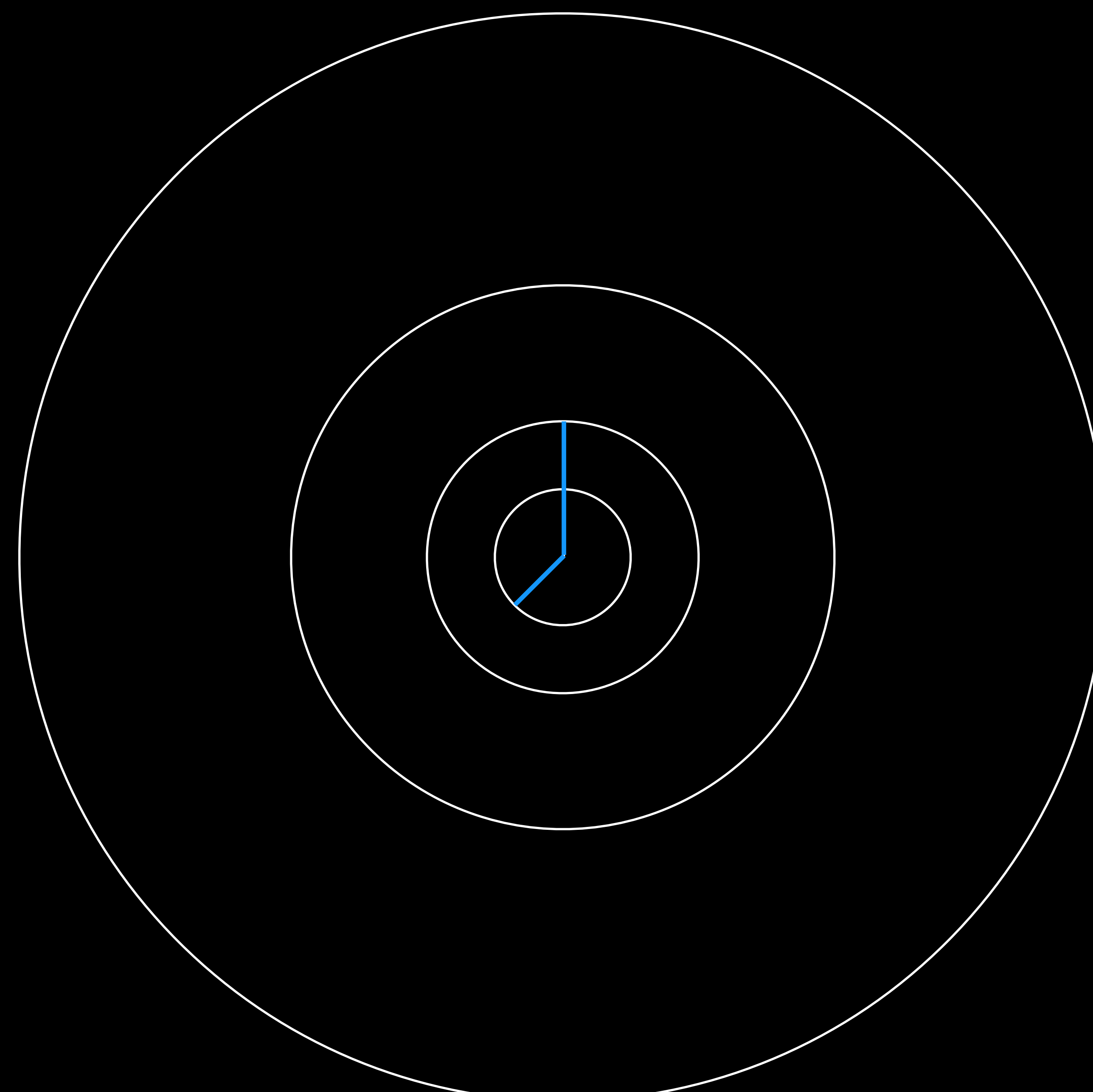

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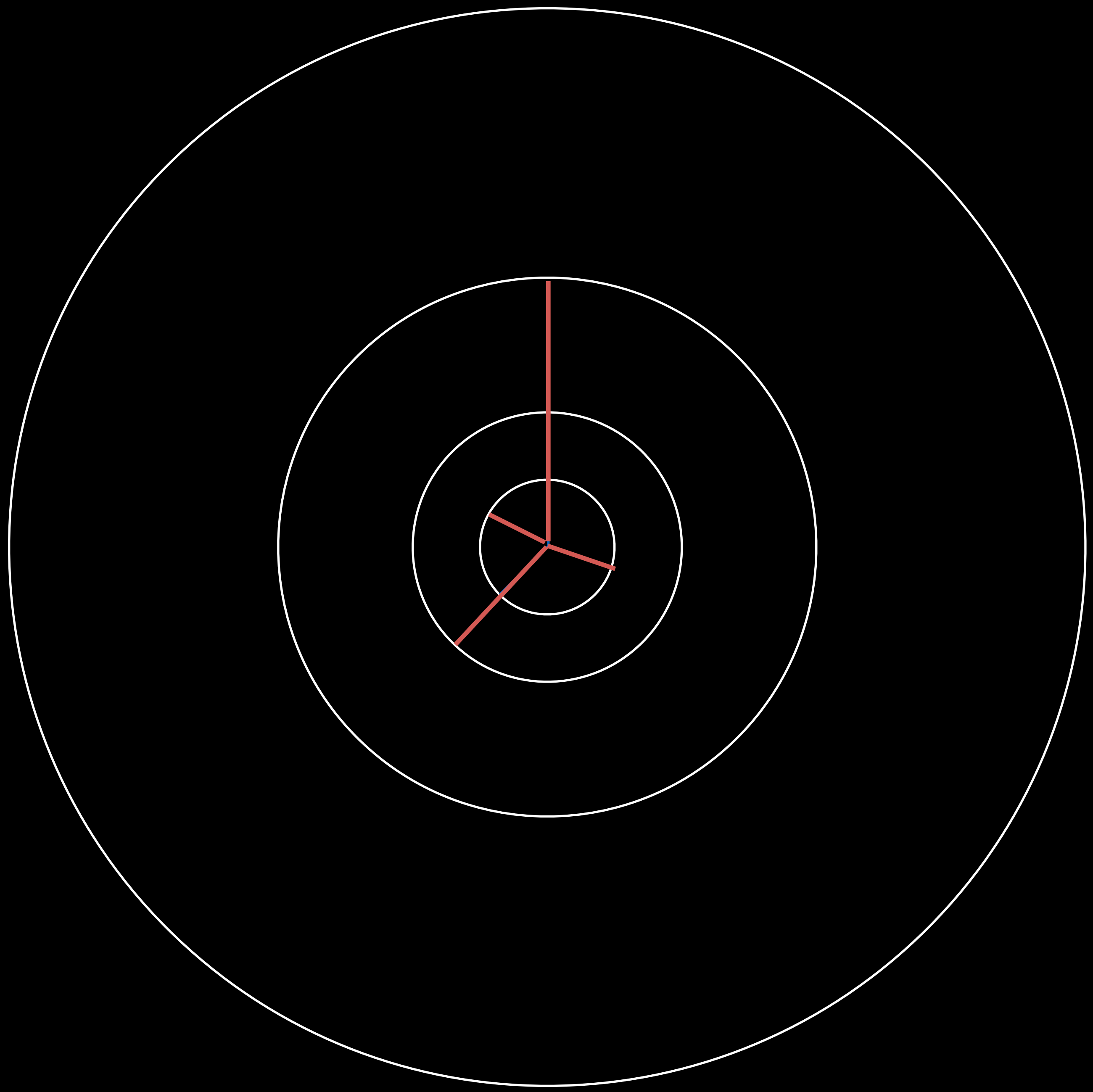

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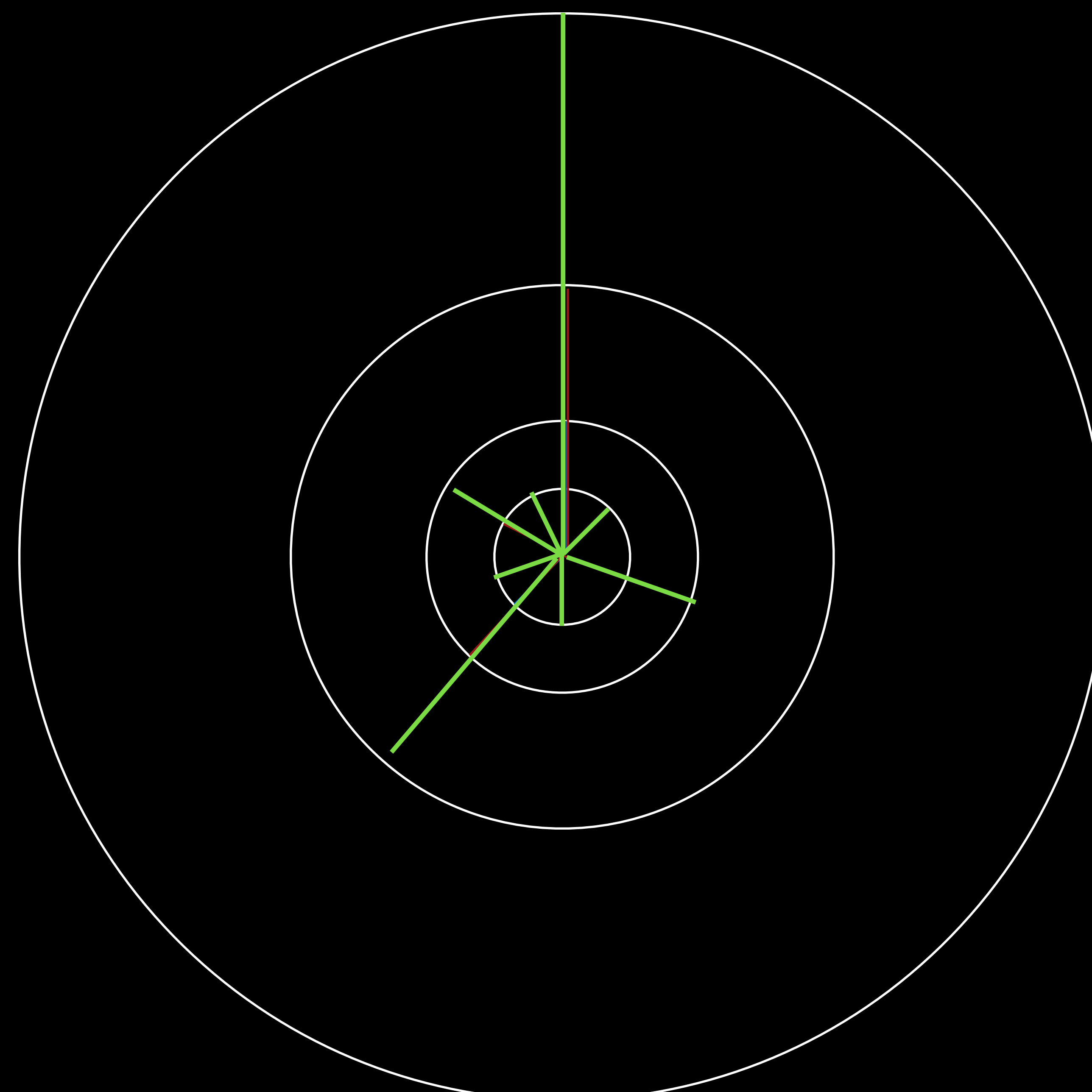

Stream Problem

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1) Target hidden in a cell

2) Algorithm chooses a stream of cells

3) Game ends when algorithm finds target

Total cost is sum of costs of cells searched

Cell (x,y) costs $x \cdot y$

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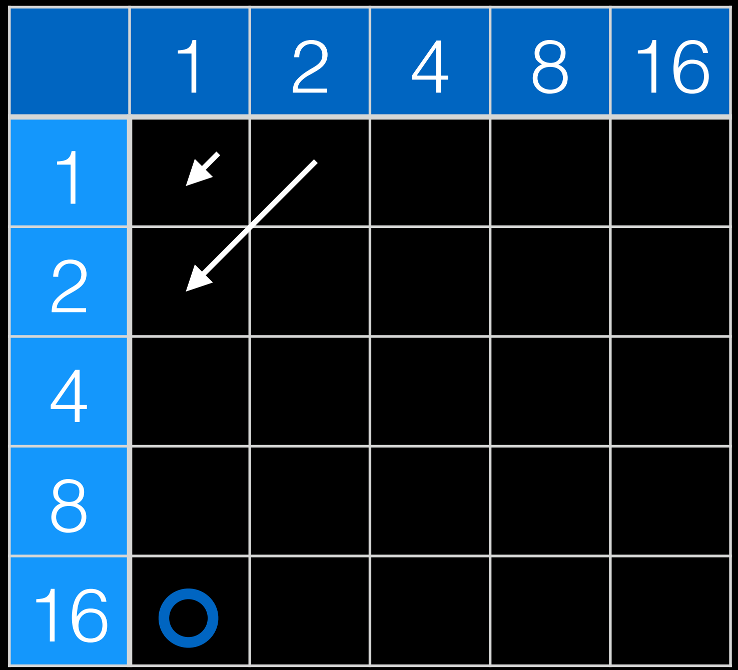

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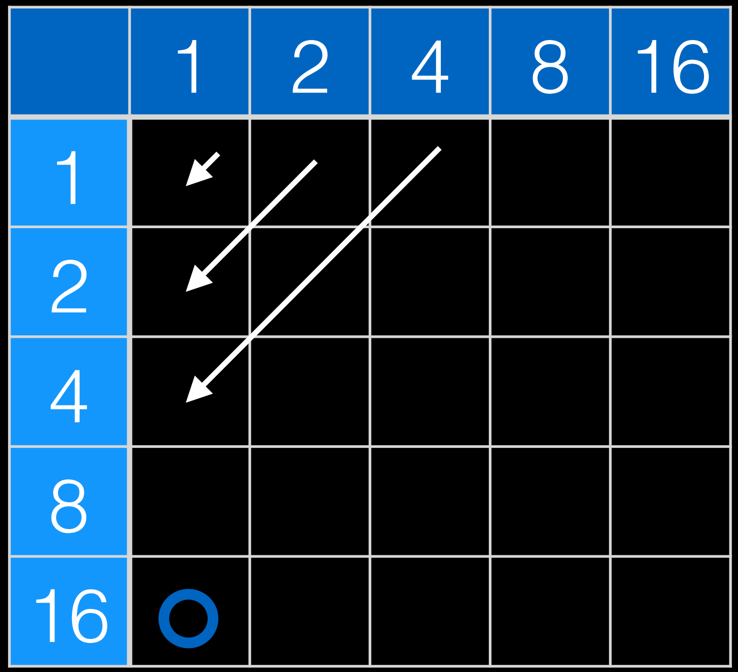

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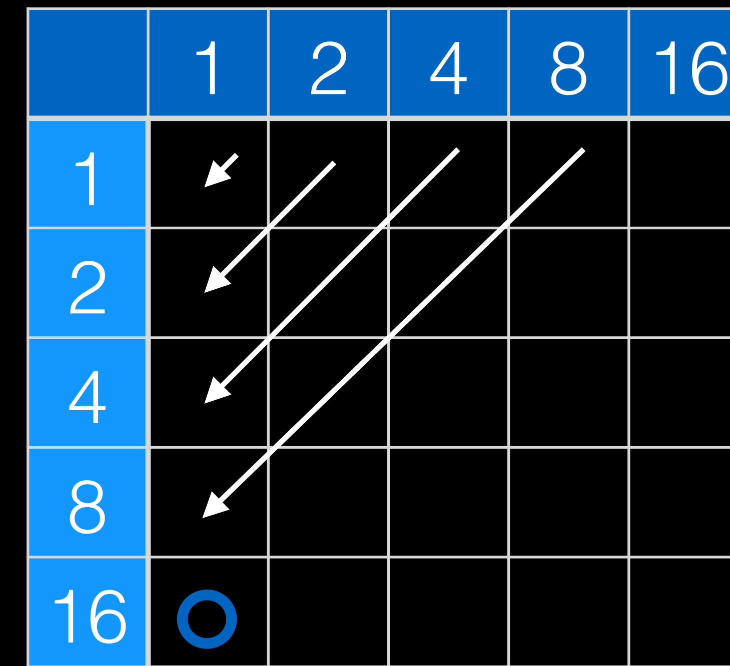

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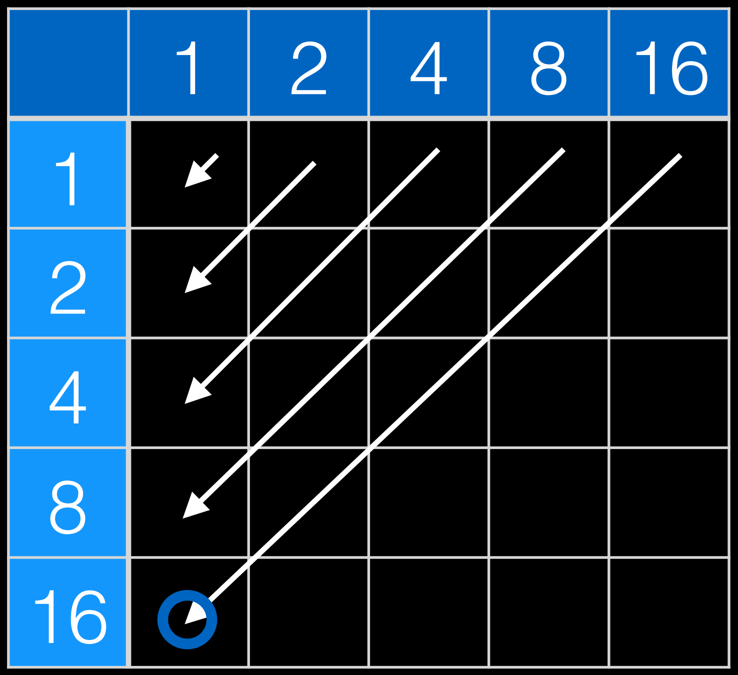

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GoldenFA

For epoch i = 1 to ∞,
  For each $1 \le x \le i$,
    Make 2^{i-x} spokes of length 2^x,
    rotated by ϕ

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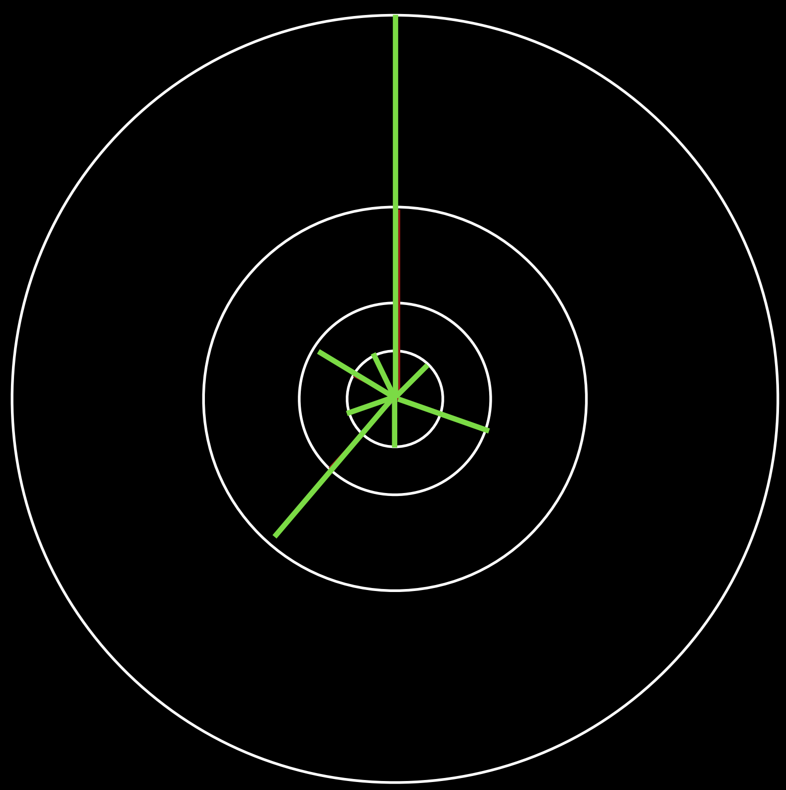

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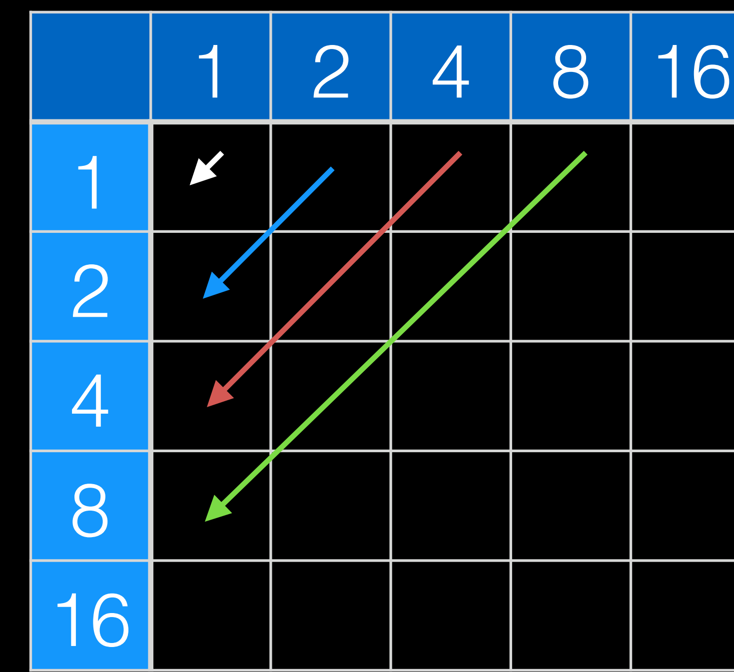
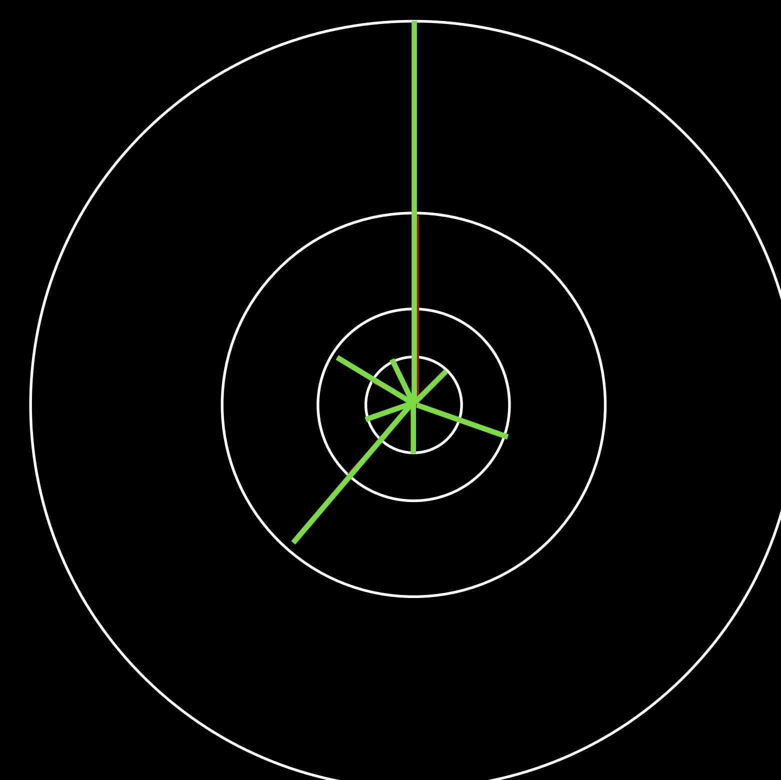


Number of epochs before reaching distance L: O(log L)



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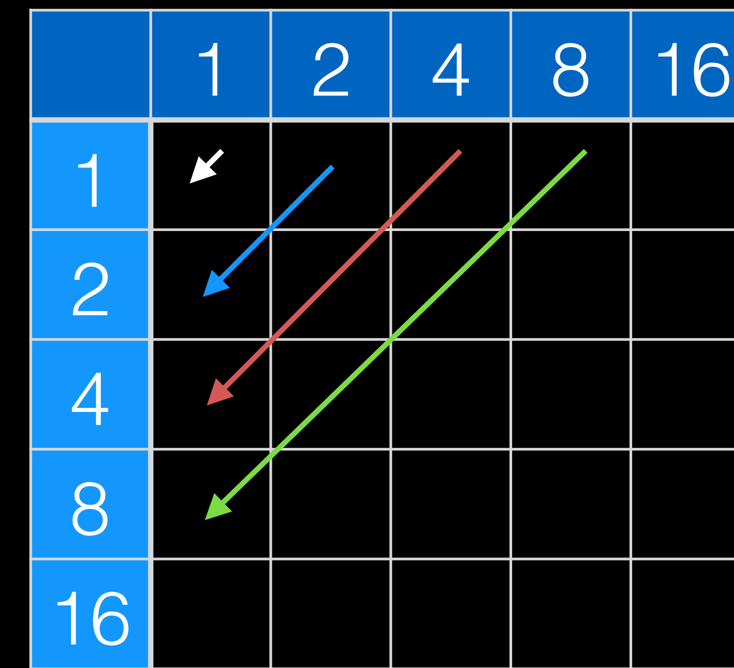

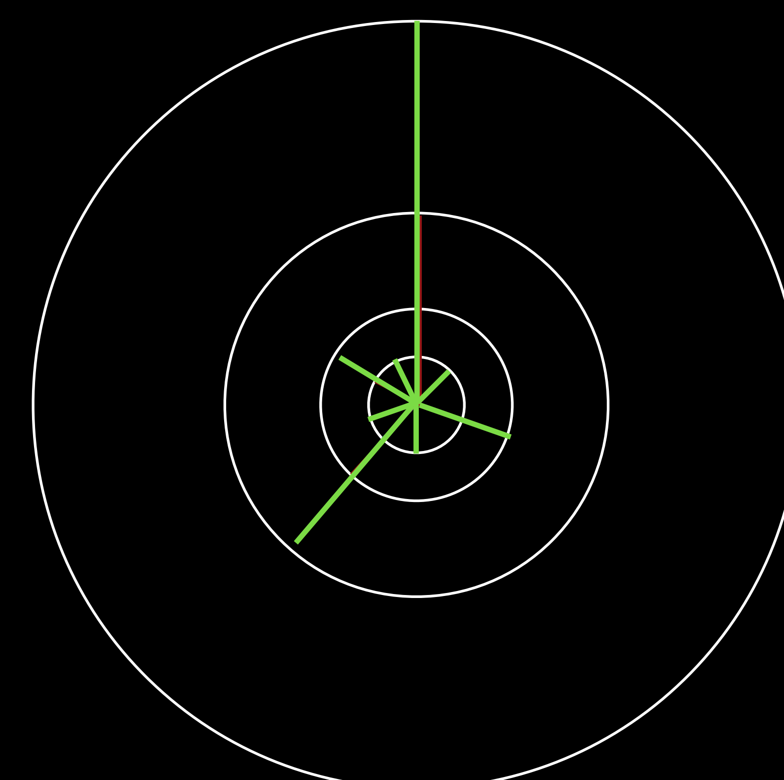

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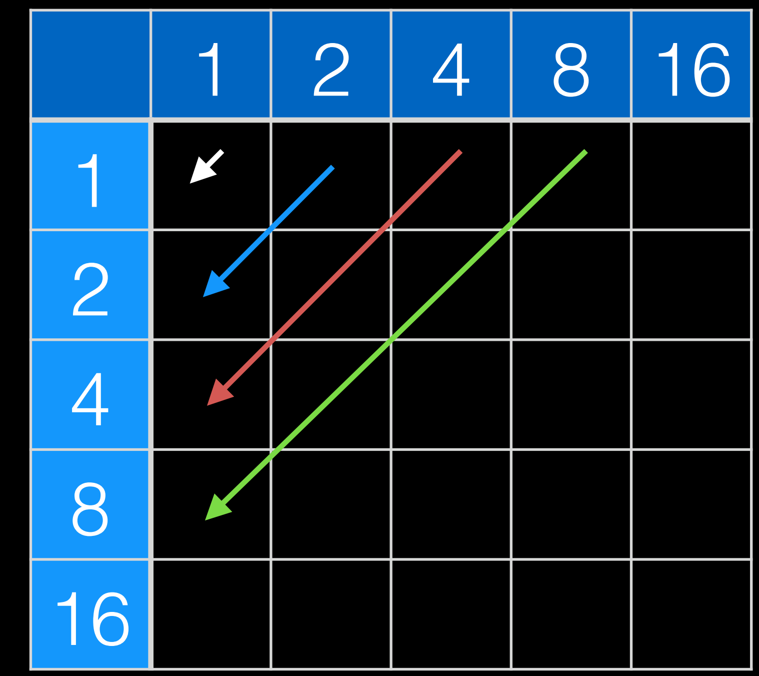
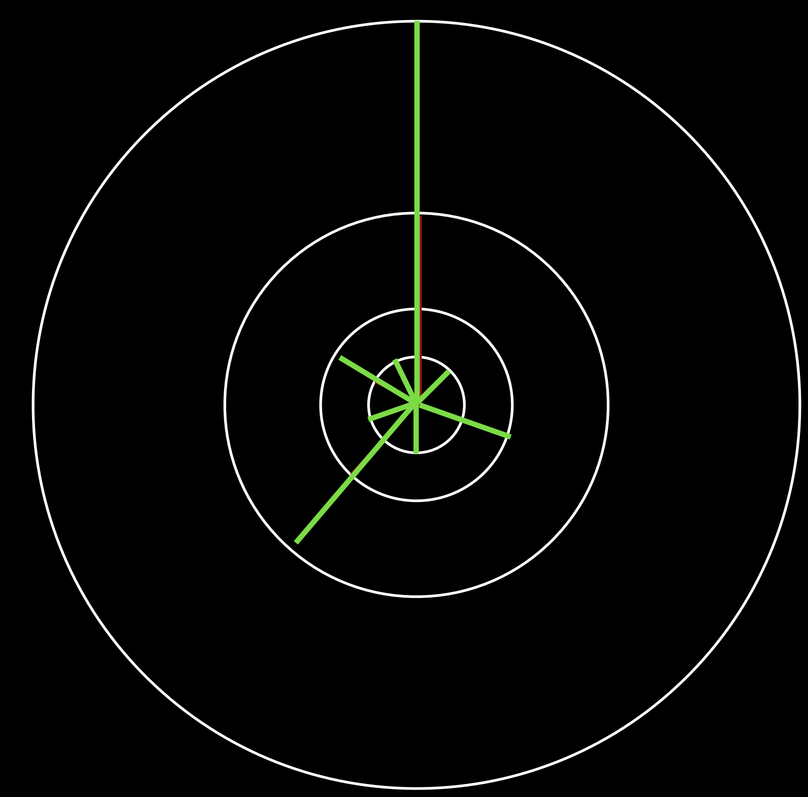


Number of epochs before reaching distance L: O(log L)

Number of epochs before the spokes are sufficiently close: O(log (L/W))



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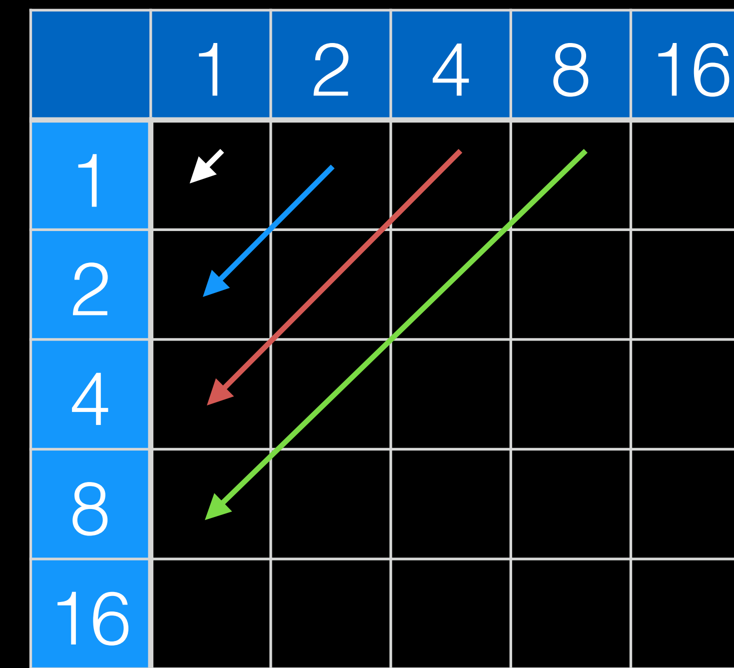

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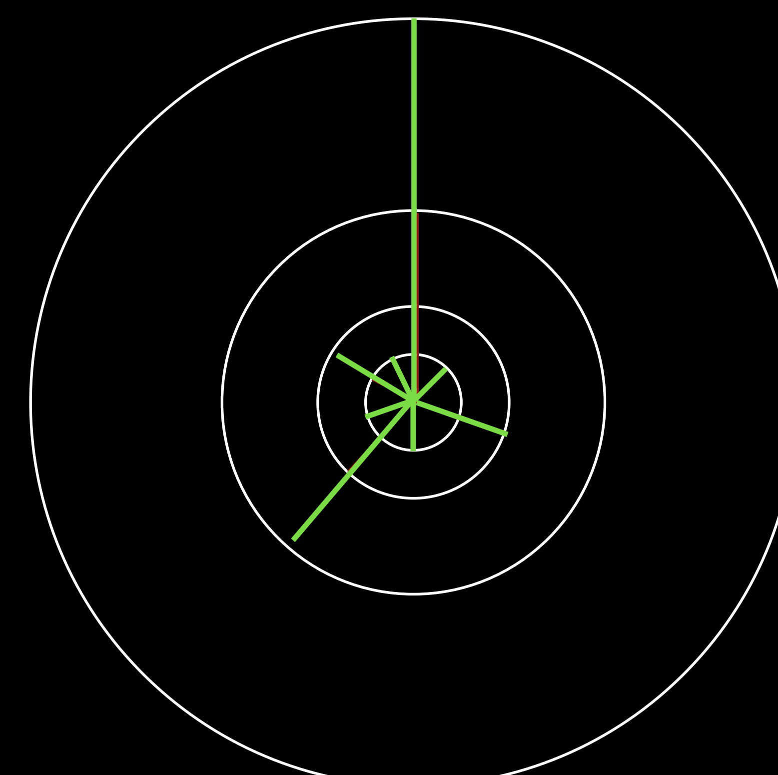

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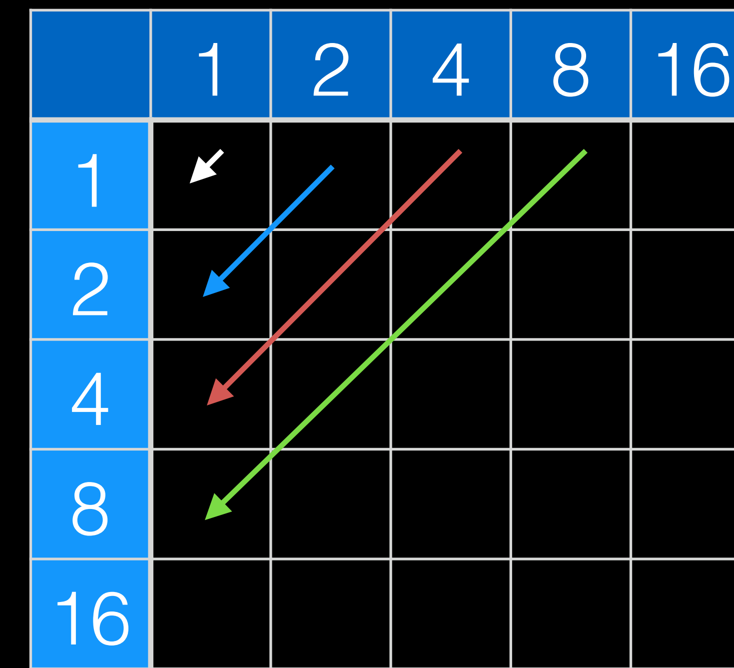

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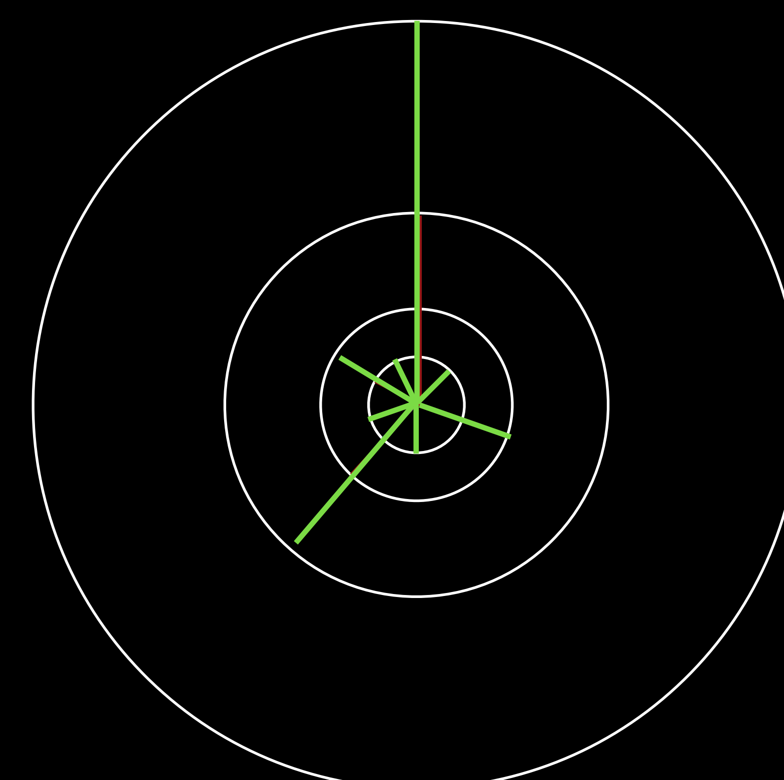
Number of epochs before the spokes are sufficiently close: O(log (L/W))

Cost for epoch i: $2^i \cdot i$



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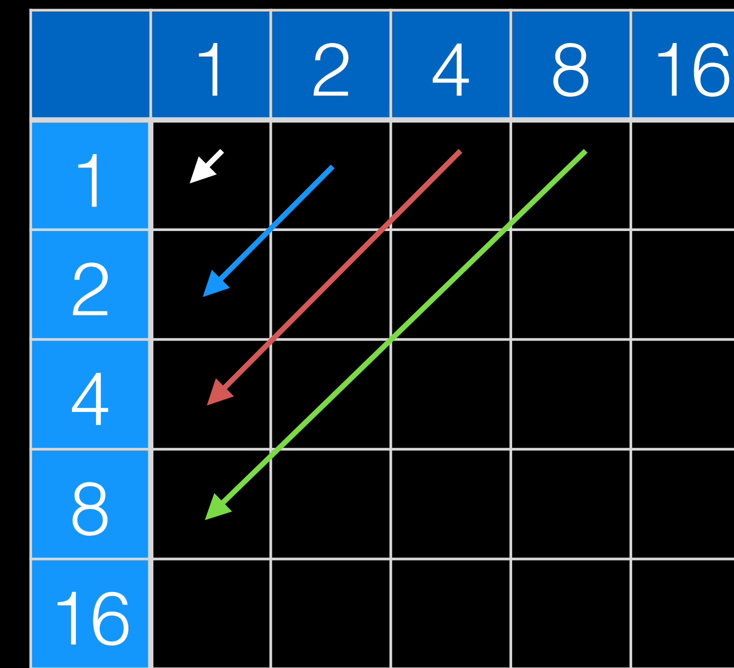

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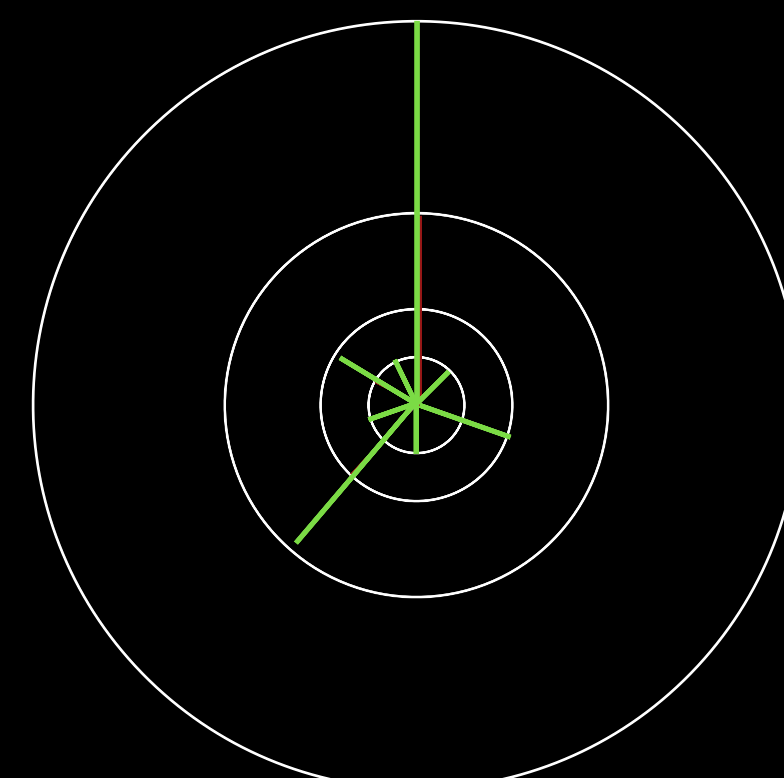

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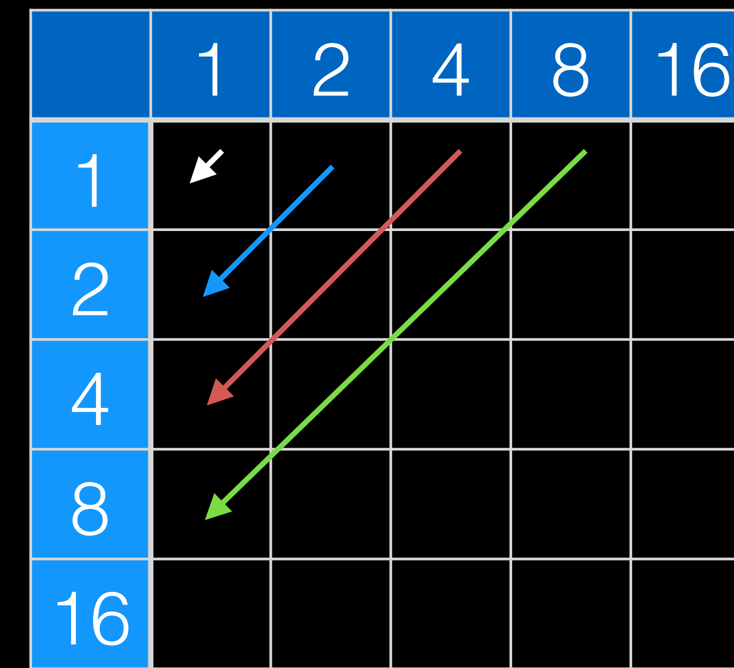

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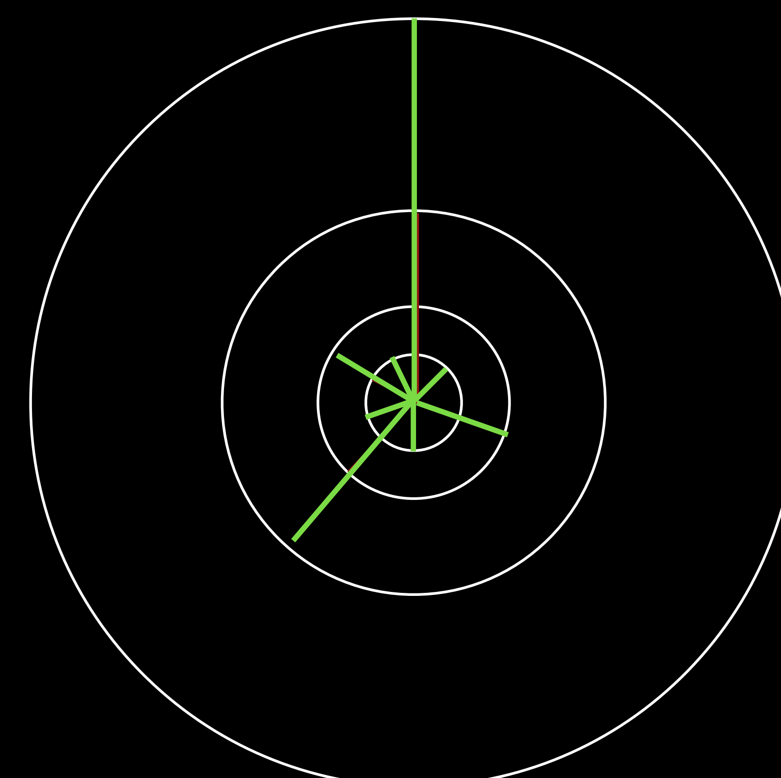
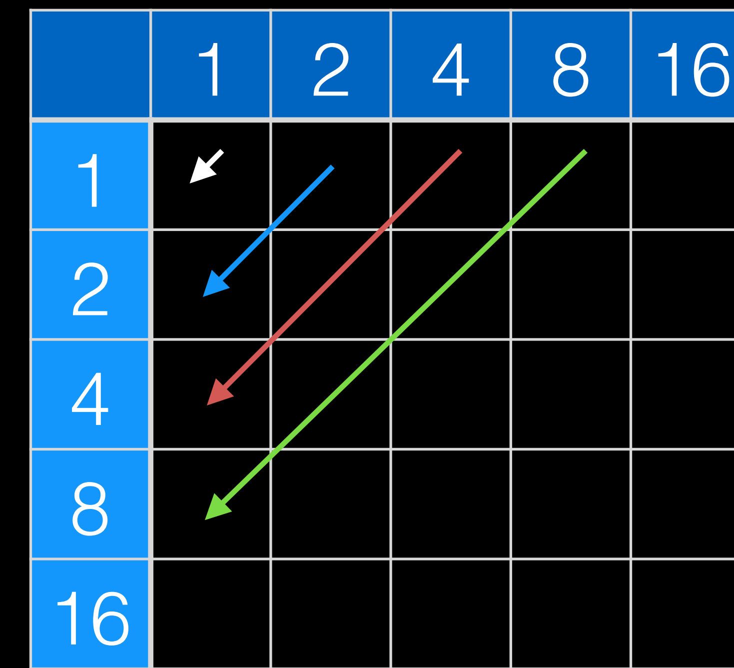
Cost for epoch i: $2^i \cdot i$

Total cost dominated by last epoch:

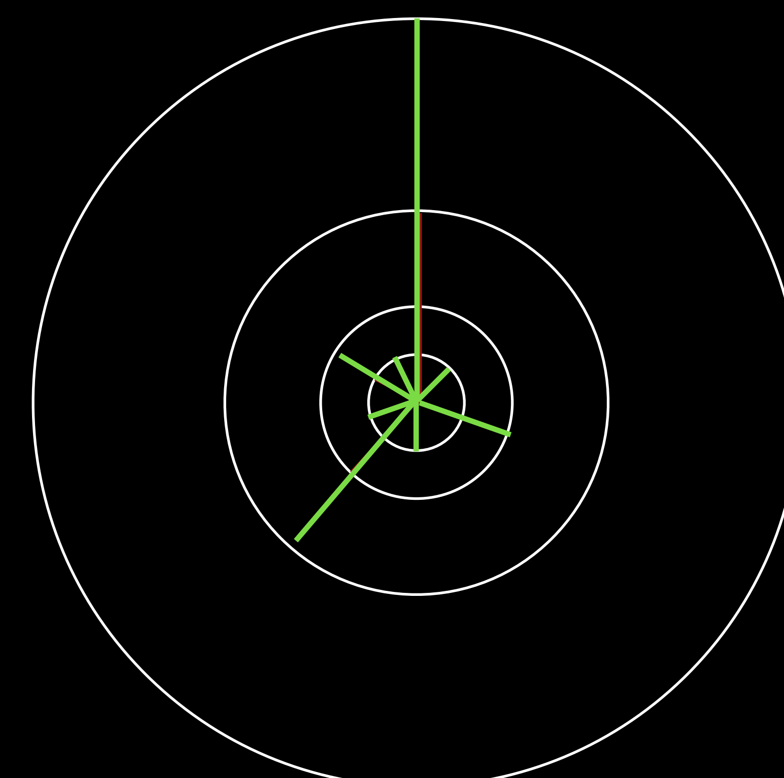


GoldenFA

For epoch i = 1 to ∞,
  For each $1 \leq x \leq i$,
    Make 2^{i-x} spokes of length 2^x,
    rotated by ϕ



Number of epochs before reaching distance L: O(log L)



Number of epochs before the spokes are sufficiently close: O(log (L/W))

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Total cost dominated by last epoch:

$$O\left(\frac{L^2}{W}\log\frac{L^2}{W}\right) = O\left(\frac{L^2}{W}\log L\right)$$

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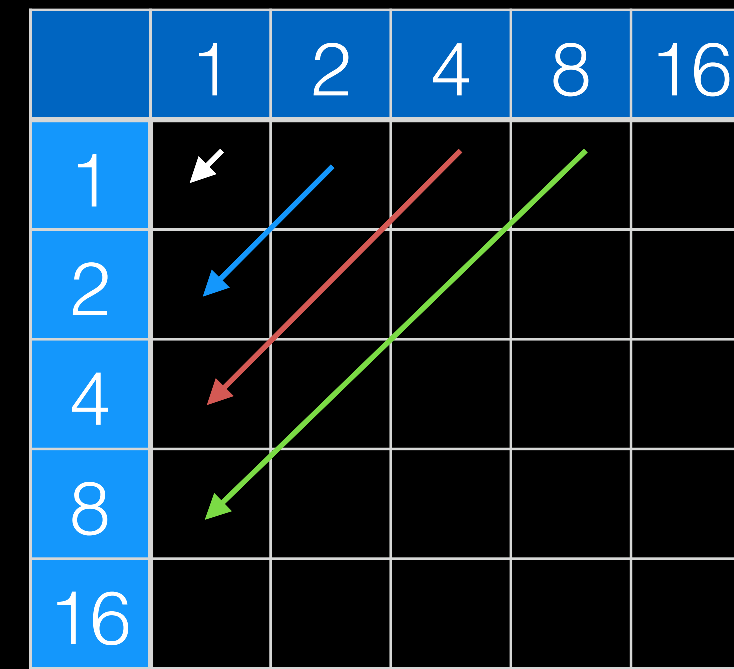

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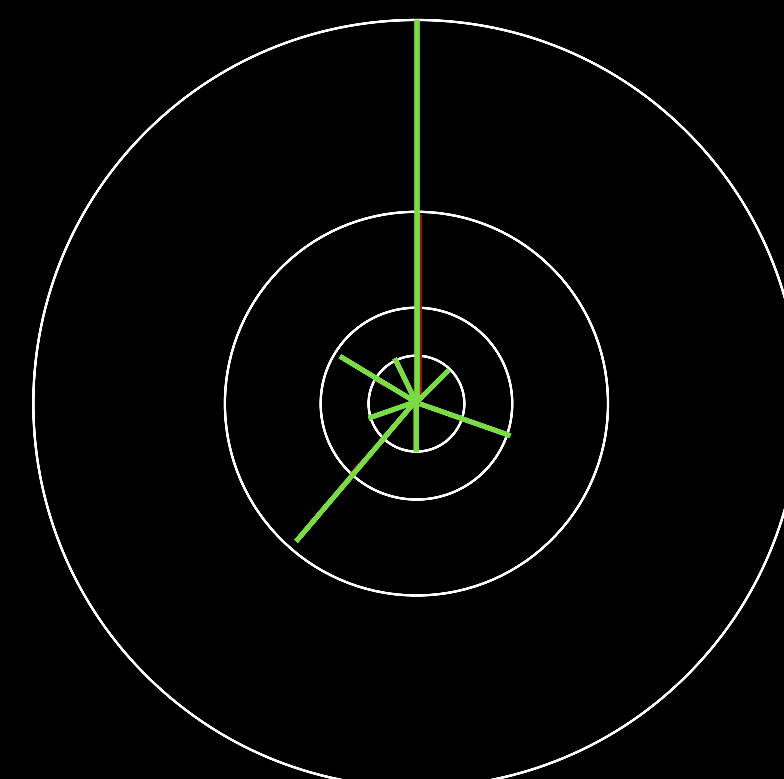

Multiple Searchers?

Multiple Searchers?

Random Initial Orientation!

N Agents

Each agent chooses a random initial heading
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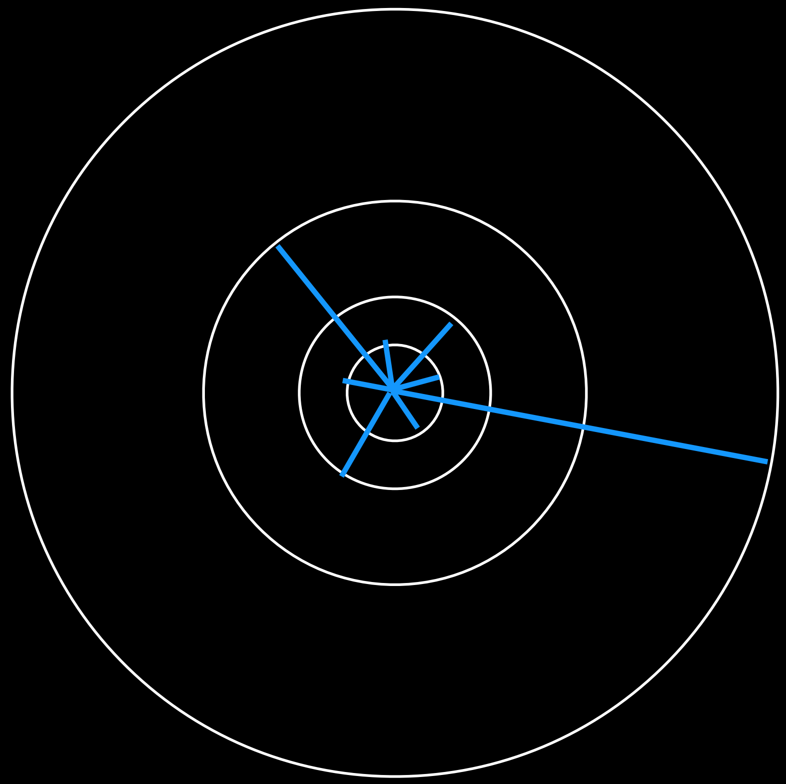
N = 3



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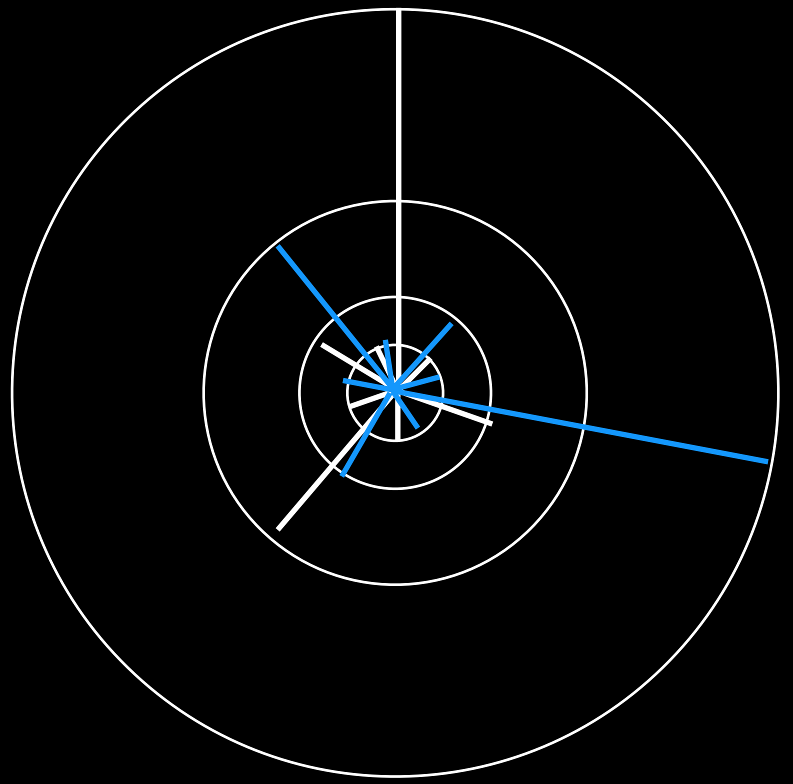

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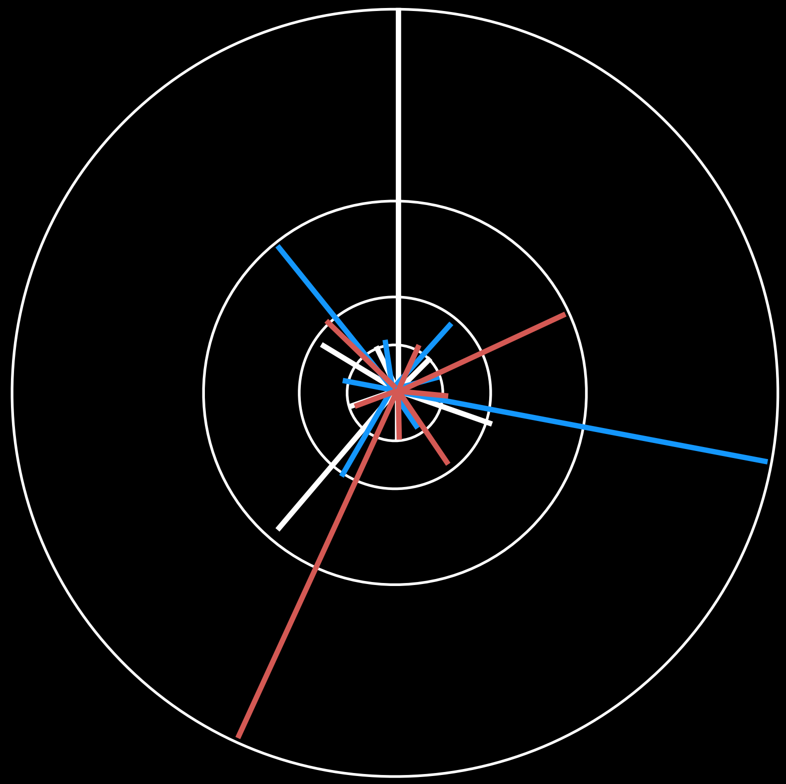

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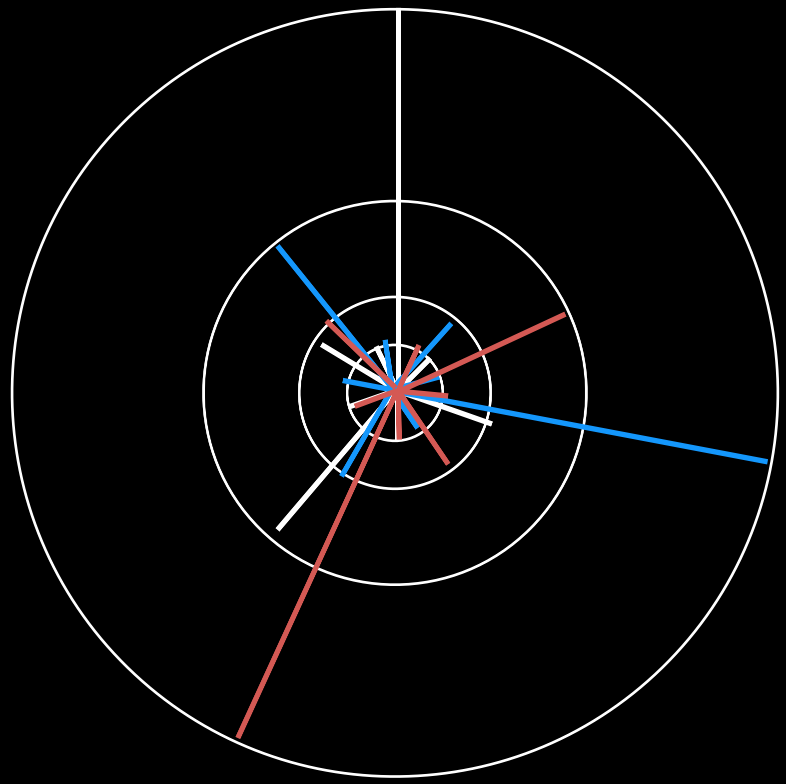

$t < N$ faults

Expected search time:

$$O\left(\left(L + \frac{L^2(t+1)}{NW}\right)\log L\right)$$

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$$\Omega\left(L + \left(\frac{L^2(t+1)}{NW}\right)\log L\right)$$

"Spoke-based": All search via line segments from nest

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Yao's Lemma
on Stream Problem

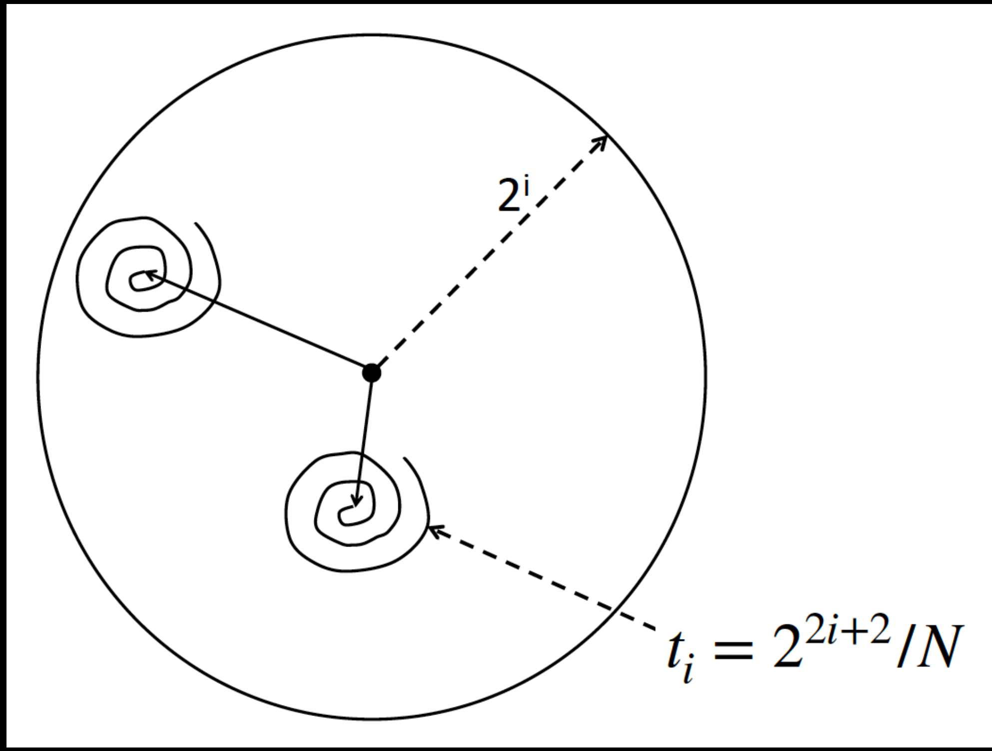
"Spoke-based": All search via line segments from nest

Experiments

F&K Advice

Each agent does the following:

For stage j = 1 to ∞
    For phase i = 1 to j
        Go to a random point at distance $\leq 2^i$
        Spiral search for time $2^{2i+2}/N$
        Return to nest



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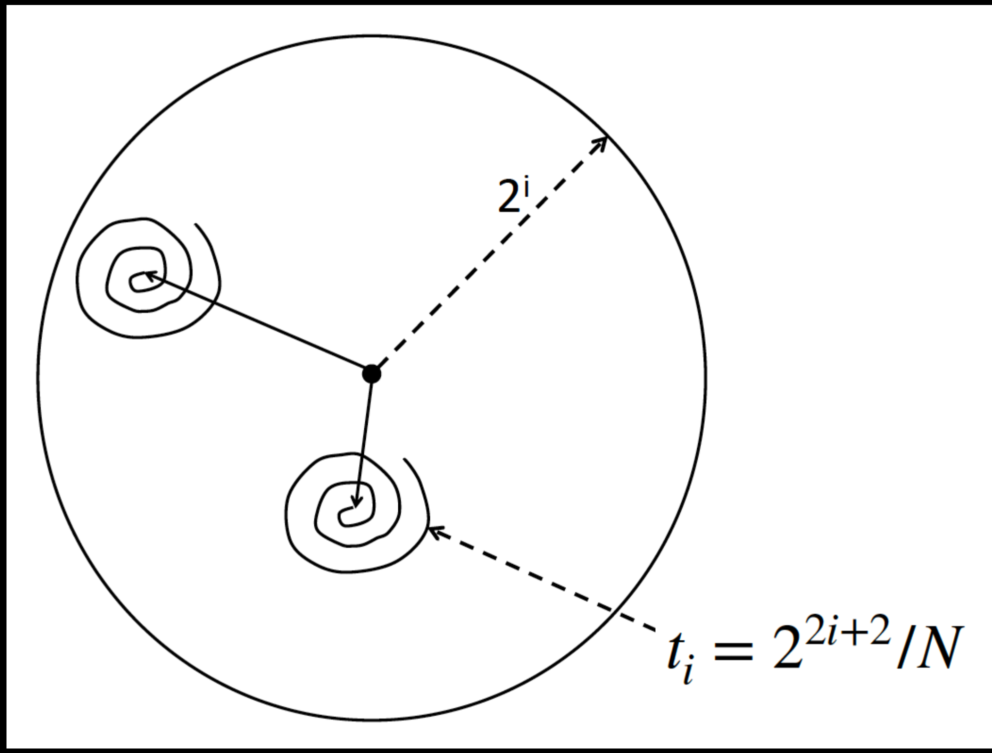

$\log N$ bits of advice to know N

$\log \log N$ bits of advice to know 2-approximation to N

F&K NoAdvice (fix $\epsilon > 0$)

Each agent does the following

For epoch $\ell = 0$ to ∞

For stage i = 0 to ℓ

For phase j = 0 to i

Go to a random point at distance $\leq \sqrt{\dfrac{2^{i+j}}{\lceil \log^{1+\epsilon} 2^j \rceil}}$

Spiral search for time $\dfrac{2^{2i+2}}{\lceil \log^{1+\epsilon} 2^j \rceil}$

Return to nest

Algorithms Tested

Algorithm	**Advice (bits)**	**Robustness**	**Runtime**
F&K (advice)	$O(\log\log N)$	Not Robust	$O\left(L + \frac{L^2}{N}\right)$ for $W = \Theta(1)$
F&K (no advice)	0	Not Robust	$O\left(\left(L + \frac{L^2}{N}\right)\log^{1+\varepsilon} N\right)$ for fixed $\varepsilon > 0$ and $W = \Theta(1)$
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GoldenFA-Heuristic:

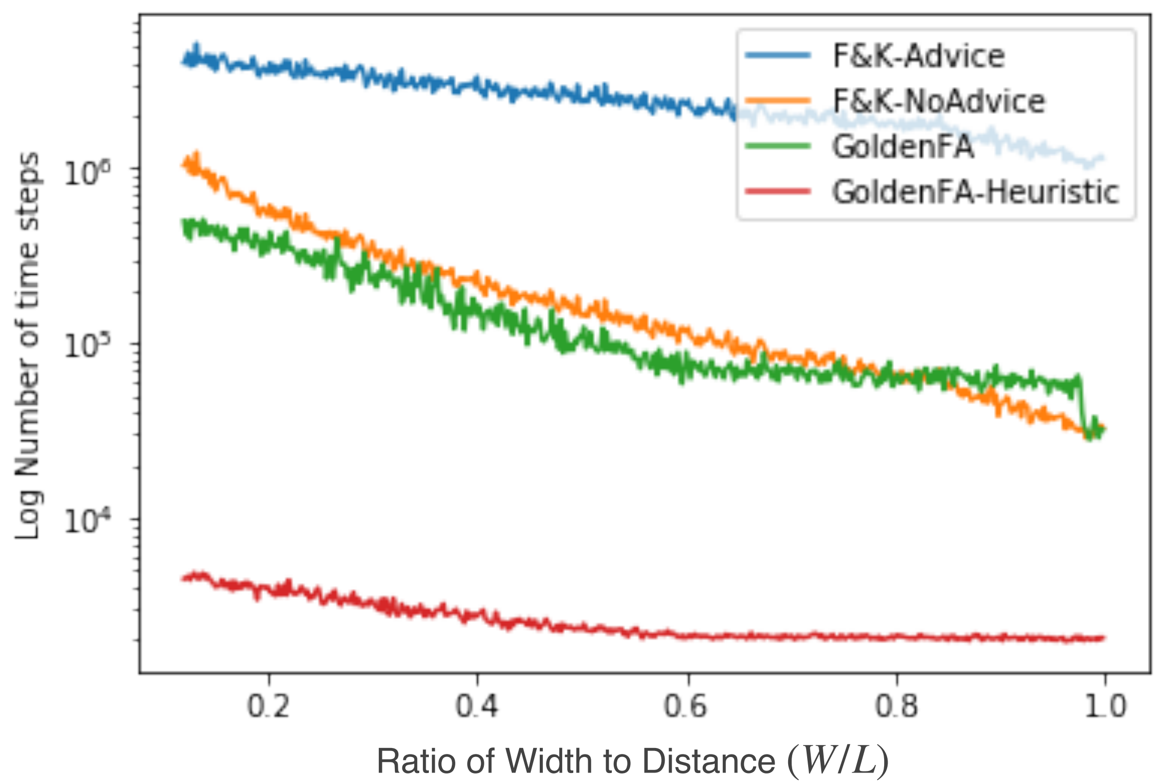
In epoch i, make $\lceil c(1+\alpha) \rceil$ spokes of length $(1+\alpha)^i$

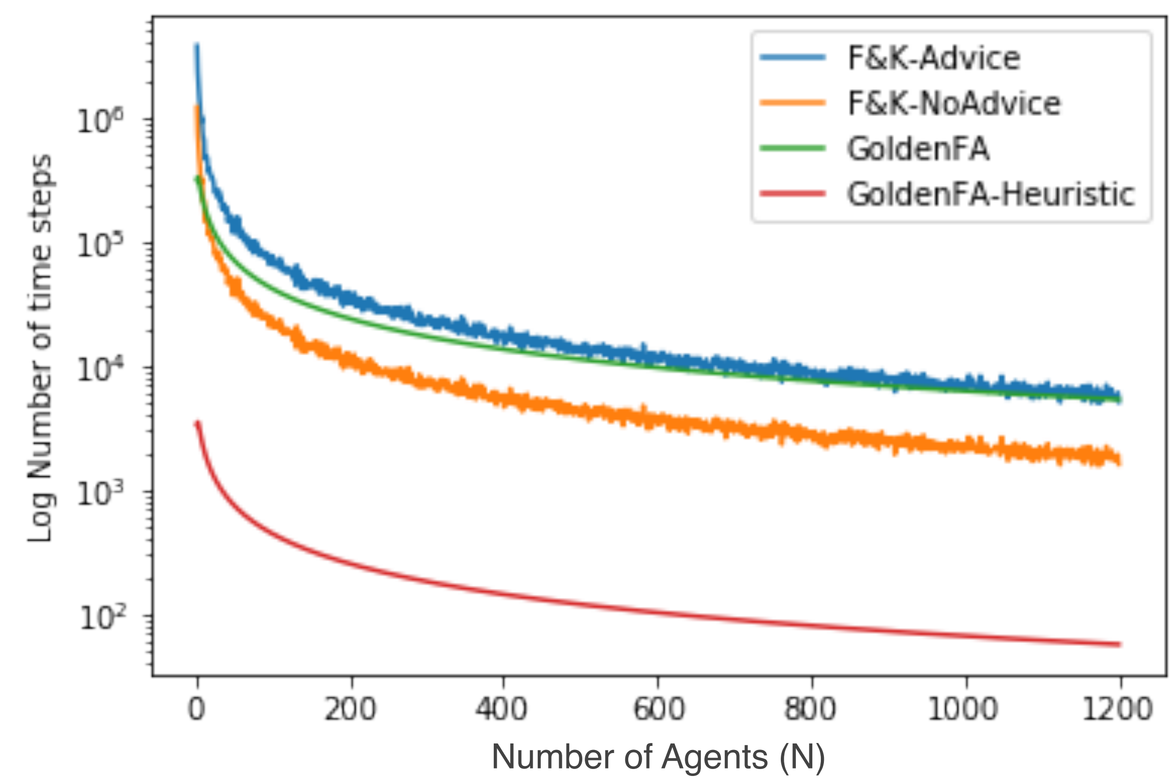
$c \leftarrow 1.9; \alpha \leftarrow 7$

F&K-NoAdvice:

$\epsilon \leftarrow .01$

Varying W; Varying N

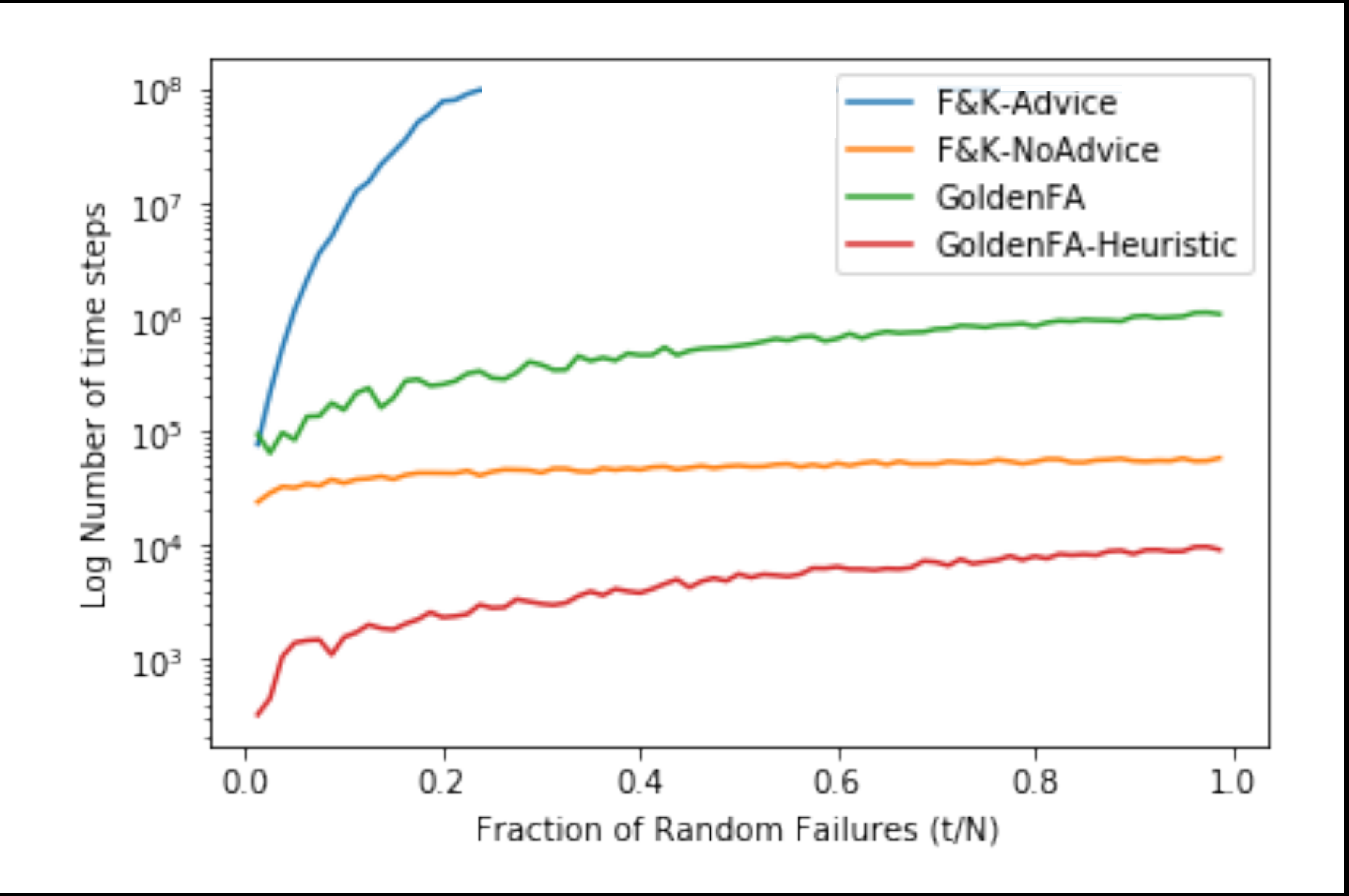




$L = 500; N = 1$

$L = 500; W = 4$

Faults



$L = 500; D = 4; N = 100$

Conclusion


THAT'S IT
I'VE HAD IT WITH THESE
%$@#!@$%!
ANTS
ON THIS
%$@#!@$%!
PLANE

Results Recap

L = target distance; W = target width;
N = # agents; t = # faults

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Future Work

Get the %$@#%! ANTS off the %$@#%! plane

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Theoretical Problem: Search in $\mathbb{R}^3$

Get the %$@#%! ANTS off the %$@#%! plane

Theoretical Problem: Search in $\mathbb{R}^3$

Practical Problem: Many searches have properties that simplify search along third dimension

Target Density

Target Density

Assume: Agent can sense local target density

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Assume: Agent can sense local target density

General Problem: Order statistics

Target Density

Assume: Agent can sense local target density

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Problem 1: Efficiently estimate target mass

Target Density

Assume: Agent can sense local target density

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Problem 1: Efficiently estimate target mass

Problem 2: Find max target density

Target Density

Assume: Agent can sense local target density

General Problem: Order statistics

Problem 1: Efficiently estimate target mass

Problem 2: Find max target density

Gradient Descent

Questions?