The Impact of Social Networks on Multi-Agent Recommender Systems

Hamilton Link[1,2], Jared Saia[1], Terran Lane[1], and Randall A. LaViolette[2]

[1] Department of Computer Science
University of New Mexico
Albuquerque, NM
[2] Sandia National Laboratories
Albuquerque, NM

Abstract. Awerbuch et al.'s approach to distributed recommender systems (DRSs) is to have agents sample products at random while randomly querying one another for the best item they have found; we improve upon this by adding a communication network. Agents can only communicate with their immediate neighbors in the network, but neighboring agents may or may not represent users with common interests. We define two network structures: in the "mailing-list model," agents representing similar users form cliques, while in the "word-of-mouth model" the agents are distributed randomly in a scale-free network (SFN). In both models, agents tell their neighbors about satisfactory products as they are found. In the word-of-mouth model, knowledge of items propagates only through interested agents, and the SFN parameters affect the system's performance. We include a summary of our new results on the character and parameters of random subgraphs of SFNs, in particular SFNs with power-law degree distributions down to minimum degree 1. These networks are not as resilient as Cohen et al. originally suggested. In the case of the widely-cited "Internet resilience" result, high failure rates actually lead to the orphaning of half of the surviving nodes after 60% of the network has failed and the complete disintegration of the network at 90%. We show that given an appropriate network, the communication network reduces the number of sampled items, the number of messages sent, and the amount of "spam." We conclude that in many cases DRSs will be useful for sharing information in a multi-agent learning system.

1 Introduction

One of the canonical problems of machine learning is recommending products to potential users, i.e., presenting each user with one or more items they are likely to be satisfied with. Recommendations can be given based on features of products, user similarity, or both, but the algorithms that learn from this data are typically centralized [6, 17]. Awerbuch et al. [5] presents what appears to be the first distributed recommendation system (DRS) algorithm, albeit with a relaxed definition; the goal is to ensure that most users are *eventually* presented with a satisfactory item. Even so, distributed recommender systems potentially have many attractive qualities, such as privacy and fault-tolerance. They avoid gathering personal information from a large number of users back to a central repository. A large system could learn how best to distribute announcements of new technical papers, for example, by locally capturing information on research interests and sharing key pieces of information only between immediate peers. Such a system is also more robust to failures than a centralized one, and it does not require corporate interest to ensure servers were maintained (there being only a peer-to-peer network or similar infrastructure to begin with). If the distributed algorithm could be designed with a suitable abstraction barrier between nodes, personalization may also be enabled, with each user's system independently learning the structure and parameters of that user's interests and how each neighbor's preferences are related.

We consider the impact of an explicit communication network in a DRS with requirements similar to [5]. For this work, agents (the DRS nodes that act on each user's behalf) were placed into a graph, and were only able to communicate with their immediate neighbors. Random polling was dropped in favor of local broadcast.[3] Given this new model, we considered the algorithm's performance under two graph structures: one in which users with substantial common interests have agents organized into cliques, and one in which the agents are randomly connected in a scale-free network [3]. With a network, the amount of work required to sample the space of products is improved, in part because broadcasts are used to spread knowledge of liked items more rapidly. The communication complexity is also greatly improved, and this includes a reduction in the amount of uninformative traffic, or "spam."

In the mailing-list model, agents with significant common interests are connected to one another by design. Such a system must be engineered to ensure that new members are properly connected. It is conceivable, however, that other network structures would naturally ensure that new members would serendipitously connect to at least one existing member with common interests, effectively causing groups of users with overlapping interests to form connected subgraphs in the population. Previously published results by Cohen et al. on the resilience of scale-free networks to random node failures [8] suggested SFNs might have this property, and this led us to use SFNs for the word-of-mouth model. Although the original resilience results do not seem to apply to SFNs of minimum degree 1, SFN structure varies with the power-law exponent used, and we have identified parameters for which a DRS would perform well.

We have a more accurate estimation of the properties of scale-free networks when a potentially large fraction of the nodes fail uniformly at random. The approximations used in the original work on this subject [8] led to the claim that the Internet[4] would retain a spanning cluster across surviving nodes even if 99% of its nodes failed. We conclude, based on more exact formulation, that the original approximations are highly optimistic when the degree distribution obeys

[3] This would have been untenable in the model of [5], which uses global interactions.

[4] The Internet is cited as a scale-free network with an exponent of 2.5 and minimum degree 1 in [8], but see also [1, 7, 20].

a power law down to a minimum degree of 1, and that the critical fraction is at best 89.8%. Furthermore, under such conditions any "spanning" component would only capture a small constant fraction of the surviving nodes – if 60% or more of the network was to fail, at least half of the remaining nodes would be neighborless orphans. We achieved this result by specifically considering the degree-1 case, explicitly considering the portion of the network that is orphaned, and approximating the resulting network with explicit size and power-law parameters as in [3].

Section 2 reviews the most relevant literature in recommender systems and cites the most relevant literature on scale-free networks; section 3 covers the distributed recommender system model from [5] in more detail and introduces notation that will be used later. Section 4 outlines our result in the conditions for resilience of scale-free networks and explains its relevance to DRSs. Sections 5 and 6 use the established notation and graph properties to derive our algorithm's performance in the number of sampled items, the number of messages sent, and the amount of "spam" sent. The final section summarizes our results and the work that remains to be done.

2 Related Work

The Internet is full of examples of centralized recommender systems (Google, Amazon, etc.). These systems are a mix of collaborative filters and content-based recommendation systems, but these systems all collect, analyze, and use the information centrally. This is perfectly legitimate, but it is not the only available strategy; in many cases the recommendations given are either the service being provided or are a customer service related to the hosting business. Centralized algorithms for recommender systems have been covered extensively in the literature [2, 18].

Distributed recommendation systems are not as highly represented. In [5] the definition of "recommendation system" is altered slightly to require that users eventually find a satisfactory item in a stream of recommendations, instead of being given a short list of the most promising items. Conceptually, we would like to design systems like [5] that use a DRS to efficiently pass information about potentially desirable items through to potential users and then locally apply a user-specific centralized content-based algorithm to filter and rank items as they arrive. This is like an idealized email system in which people only send messages within their social circle, and in which all users have a learning mail filter such as PopFile [13]. From the centralized point of view the most analogous work we have found is content-boosted collaborative filtering (CBCF) [17].

Scale-free networks were originally of interest to us because of their published resilience to random failures [8, 22], which implied random subgraphs of an SFN had a good chance of being highly connected (subject to the power-law exponent of the original graph). This suggests that simply propagating items through the network from their point of discovery, in a manner similar to [5], would with high probability reach most or all of the agents that would be interested in the item. Recent SFN literature can be roughly divided into theoretical and empirical camps. The formal treatments are based in physics, statistical mechanics, and mathematics [3, 4, 8, 19] and describe the mathematical properties that can be derived from the assumption that SFN node degrees follow a power-law distribution. The study-driven work [10, 21, 22] is aimed at capturing or sampling the structure and degree distribution of real-world networks such as electric power transmission, Internet routing, web pages, social networks, etc. in order to see if the observed systems are scale free, and to verify the theoretical properties. Many authors refer to the tendency of non-engineered Internet communities to form scale-free networks, although for the Internet some of this work has used potentially biased forms of sampling [1].

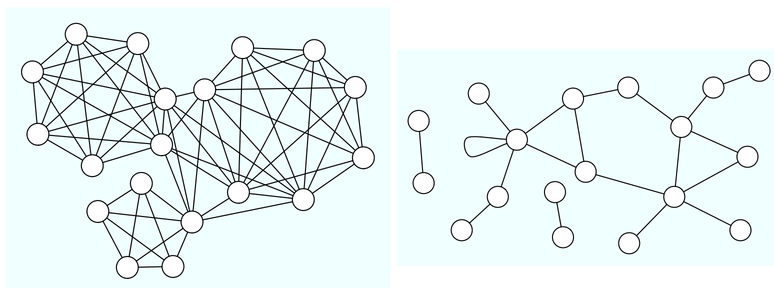
The viral spread of information (or pathogens) in social and information networks has also been studied extensively [14], and the relationship between recommender systems and epidemic spread in subpopulations of an SFN is of interest for further study. Another closely related subdomain of graph theory focuses on "small-world" networks [23]. We have not analyzed a DRS overlaid on these graphs but this would be an obvious possible extension to our work.

3 Distributed Recommender Systems

In [5] users search for items they like in some set of products. They alternate between sampling the set of all products and asking other users for recommendations. The *raison d'être* of these users is to try products in P, find things they like, and send messages about good items to the other members of their "special interest group" (SIG). In a slightly less abstract world, our users have software agents acting on their behalf in a network. The agents must learn their user's preferences, search P and solicit their user's opinion on items, and forward messages to one another.

Assume we have a set of agents U of size μ, and a set of products P of size η, with each user $u \in U$ having a predetermined but unknown set of products they will like, $P(u)$. It is reasonable to assume that SIG members have more in common with one another than a member's fraction of desirable products in P, i.e. $\frac{P(u)}{\eta} < \frac{P(S)}{P(v)}$. If this were not the case, items recommended by other users would have a lower expectation of satisfying the recipient than something chosen at random. Let there be a set of special interest groups $\mathcal{S} = \{S : S \subset U, \bigcap_{u \in S} P(u) \neq \phi\}$, where each SIG S is a set of users with common interests. As in [5], the stated goal is that "a large fraction λ of the users will find a good product in the set recommended to them" given that "there exists a small collection of SIGs that cover most users," i.e. the fraction of U represented by all of $\mathcal{S}$ is at least λ and the number of SIGs $\ell = |\mathcal{S}|$ is of order $\Theta(1)$. In [5] there is no requirement that the set of products recommended to an individual be small, but it should be clear that this would be desirable.

For this work, we add a graph G representing the communication network available to the users. In the "mailing-list model," users are organized into (potentially overlapping) cliques based on their SIGs (Figure 3a). In the "word-of-mouth model," the users are randomly distributed in a scale-free network (Figure 3b). In either model, we can consider the subgraph G' comprising some set of vertices in G (corresponding to the members of a SIG). Users communicate by broadcasting positive findings to their immediate neighbors in the graph. When a user u receives a product name from v, they sample the item, and may or may not generate further messages (depending on the structure of the network) if it is satisfactory. For our analysis, users sample shared items independently of their continuous random sampling.



(a) (b)

Fig. 1. The mailing-list model (a) with four SIGs sizes 5, 7, 7, and 6, and the word of mouth model (b) generated with a random configuration given a power-law degree distribution with $\alpha = 2.3$ and $\beta = 1.2$. Notice the presence of a self-arc in the latter. Self-arcs are an inevitable artifact of generating simulated networks using the random configuration model but occur in no significant fraction in large graphs.

A related question to the amount of work required to disseminate popular items (and the portion of this that leads to unfruitful probes of products, if probes have an associated cost) is the amount of unproductive traffic, or "spam", generated during the cooperative process. In the mailing-list model and word-of-mouth model, spam consists of an item being reported to any individual for whom that item is not in their set of interests. For the algorithm in [5] we consider spam to be any request for a recommendation sent to an unsatisfied individual, although this definition could be extended to include other unproductive traffic.

It is important to stress that this decentralized system is not solving the same problem as traditional recommendation systems; this system is not identifying or exploiting similarities between users or trying to estimate the likelihood that some user will like some item. Instead, the goal is to ensure that information about some item will *reach* most of the users that are likely to be satisfied by the item. Then, a content-based algorithm would be used to estimate how desirable the item is for each the users it reaches. CBCF algorithms are a solution of interest, if we are able to decentralize the learning problem and exploit existing social structures along with domain information.

4 Scale-Free Networks

A frequently cited result in scale-free networks is the incredible resilience to random failures of "the Internet," an example of an SFN with fairly low minimum degrees [8]. A corollary of that result is that small random subgraphs of such an SFN would exhibit a giant component. This would be useful in a DRS setting such as the word-of-mouth model: disinterest in an item may be seen as node failure, and the persistence of a giant component would imply the remaining SIG is able to pass information. Unfortunately, the resilience result is based on approximations that do not appear to hold when the minimum degree of the SFN is 1 (as in the Internet). To answer performance questions related to DRSs in SFNs of minimum degree 1, we determined the conditions under which random SFN subgraphs remain connected. The full derivation can be found in draft form as [16].

Random subpopulations of a scale-free network have a degree distribution that can be roughly estimated with a power law. In fact, the distribution starts with a minimum degree of 1 and is very nearly log/log linear, but with a rolloff that underrepresents high-degree nodes. The rolloff causes potential giant components in an SFN to disintegrate more readily, as shown in [9, 11], so using a pure power law in our derivations leads to an upper bound result on the critical failure rate p – where $(1-p)$ is the percentage size of the subpopulation of interest – at which point the giant component ceases to exist, given the initial graph's power law parameter β (Figure 4). This gives a lower bound on the size of SIGs necessary in the word-of-mouth model given β; our analysis also shows the relationship of β and p to the portion λ of the population that must eventually be satisfied in the DRS problem.

The number of nodes that retain no neighbors and the the number of nodes retaining only one neighbor are, respectively,

$$\#'(0) = (1-p)e^{\alpha}\chi, \qquad \chi \overset{\text{def}}{=} \sum_{k_0=1}^{e^{\frac{\alpha}{\beta}}} \frac{1}{k_0^{\beta}} p^{k_0}, \text{ and}$$

$$\#'(1) = (1-p)e^{\alpha}\xi, \quad \xi \overset{\text{def}}{=} \sum_{k_0=1}^{e^{\frac{\alpha}{\beta}}} \frac{1}{k_0^{\beta}} k_0 (1-p) p^{k_0-1}.$$

For networks with strict power-law degree distributions and minimum degree 1, there is no spanning component in the remaining graph when the slope increases beyond $\beta_0 = 3.47875$ [3]. The key result is that the slope of the degree distribution of the subgraph is greater than the slope of its parent's – putting it closer to or even beyond this threshold. The new slope β' is a function of the original slope β and the failure rate p (captured in the sums ξ and χ above):

$$\beta' = \zeta^{-1}\left(\frac{\zeta(\beta)-\chi}{\xi}\right).$$

By varying β in an engineered network, we can ensure random subgraphs of a given fractional size will be highly connected. In this way, β can be used to design a DRS network where SIGs of a given size may propagate recommendations of

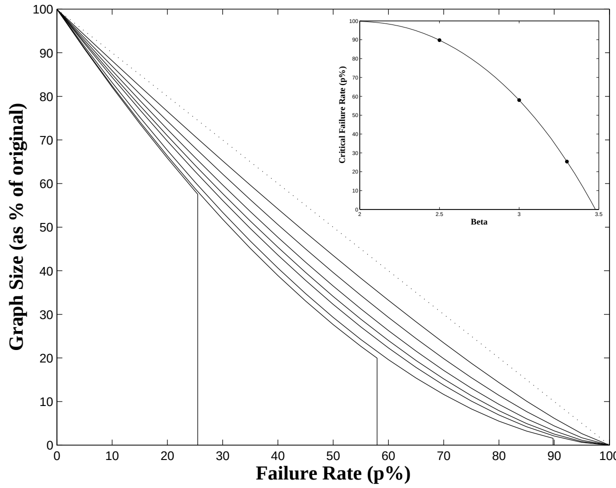


Fig. 2. The size of a SIG's subgraph (minus orphans), relative to its host SFN, as a function of the failure rate p. The distance between each curve and the diagonal $(1-p)$ equals the fraction of orphans $\#'(0)$. Curves are for $\beta = 1.2$, 1.4, 1.6, 1.8, 2.0, 2.5, 3.0, and 3.3. When $2 < \beta < \beta_0$ there is a significant critical failure rate (inset) due to the increase in β'; the critical points for the curves of $\beta = 2.5$, 3.0, and 3.3 are highlighted. For $\beta' < 2$ virtually all of the remaining graph is in the giant cluster, and this fraction declines from 90% to virtually nothing as β' goes from 2 to β_0

particular interest. In contrast, for typical values of β in empirically studied SFNs such as the Internet, a SIG comprising a small randomly-distributed group of DRS members has little chance of being able to share information without the assistance of agents outside the SIG. In addition, when the properties of the network are known and fixed, this result can be used to control the likelihood with which agents forward uninteresting recommendations, artificially boosting the effective SIG and ensuring that disconnected agents with common interests can still reach one another.

5 The Mailing-List Model

Awerbuch's model provides a fundamental theoretical result [5], but in practical terms people operate within the auspices of social circles that can provide a more structured (but still distributed) approach to finding items of interest. One possible model is that users interested in certain kinds of items will subscribe to "mailing lists" so that they can share findings with one another deliberately. While still oversimplified, this model gives additional structure and lends itself to some initial analysis. Given the users and their SIGs, we can construct a graph of user associations. In the "mailing-list model" each SIG is represented by a clique. Let G be the graph (U, E) such that $\forall\ S \in \mathcal{S}, \forall\ u, v \in S, (u, v) \in E$. When a user broadcasts a finding to all his neighbors, this is tantamount to sending an email to all of the mailing lists to which they subscribe; given the construction of G this will reach the members of all of a user's SIGs, so no retransmissions are necessary.

As in [5], consider a sequence of samples, $\sigma = \pi_1\pi_2...\pi_n$, made by the members of a lone SIG S. The expected number of samples $\mathrm{E}[n]$ before all SIG members are satisfied is at worst the number of messages generated before they find an item in $P(S)$. This is equivalent to a sequence of Bernoulli trials, giving $\mathrm{E}[n] = \eta/|P(S)|$, with each SIG member making $\frac{\eta}{|P(S)||S|}$ samples. For the last element of the sequence it takes $|S|$ messages to broadcast the finding, but some number of messages that do not satisfy the entire SIG may also be announced in the process. Each user's samples prior to this are either in $P(u)\backslash P(S)$ or $P\backslash P(u)$, with only the former generating a broadcast. In a sequence of Poisson trials with each user u having a potentially different chance $|P(u)|/\eta$ to find an item they like in a single trial, the expected number of liked items found will be tightly bounded around $\frac{\mathrm{avg}_{u\in S}|P(u)|}{\eta}\frac{\eta}{|P(S)|} = \frac{\mathrm{avg}_{u\in S}|P(u)|}{|P(S)|}$. Given this, the sample complexity[5] of the system is at most

$$C = \ell\left(\frac{\eta}{\min_i\{|P(S_i)|\}} + \mathrm{avg}_i\left\{|S_i|\frac{\mathrm{avg}_{u\in S_i}|P(u)|}{|P(S_i)|}\right\}\right).$$

This captures the samples taken by each SIG to find some liked item, and the number of extra samples users draw based on posted items. When users belong to more than one clique individual SIG interests $P(S)$ can be expected to vary in size, and the sequence σ to vary in length as a result, but the number of samples an individual user takes is simply the appropriate number for the worst-case SIG. This is because all users are broadcasting into all their SIGs, with each SIG deciding somewhat independently whether the item is of interest. Individual users' interests remain constant through this process and so the number of broadcasts they are expected to make also remains the same from the point of view of each SIG.

The mailing-list model improves upon the distributed algorithm in [5], which has a sample complexity of $2\ell\left(\frac{\eta}{\min_i\{|P(S_i)|\}} + \mu\ln(\max_i\{|U(S_i)|\})\right)$, and can be approximated as $O(\ell(\eta+\mu\ln\mu))$. In comparison $C = O(\ell(\eta+\mu))$ with the additional savings of the hidden constant factor of 2. The savings in samples required caused by the elimination of the original algorithm's sample/query alternation, which causes samples to be taken long after a suitable item for a SIG has been discovered, in favor of a network which dramatically speeds the propagation of items when found.

In addition to the modest reduction in samples taken, the mailing-list model has lower communication complexity than the original distributed algorithm, generates less unproductive network traffic, and does not assume the availability

[5] [5] defines recommendation complexity as the total number of times users test recommended products. We use sample complexity for the number of time users test products, and reserve recommendation complexity for the number of times users test *recommended* products.

of global communication. The algorithm in [5] communicates at every other step and has communication complexity $\ell\left(\frac{\eta}{\min_i\{|P(S_i)|\}} + \mu \ln(\max_i\{|U(S_i)|\})\right)$. In contrast, the mailing-list model only communicates when items of possible interest are found, leading to a communication complexity of

$$\ell \ \mathrm{avg}_i\left(|S_i|\frac{\mathrm{avg}_{u\in S_i}|P(u)|}{|P(S_i)|}\right).$$

This is far less than the distributed algorithm and is independent of the size of the set of objects, which is particularly important when $|P(u)| \ll \eta$.

Of the traffic generated, a portion of it is "spam" from the recipient's point of view. The spam generated in the mailing-list model consists of personal interests broadcast, totalling

$$\ell \ \mathrm{avg}_i\left(|S_i|\frac{\mathrm{avg}_{u\in S_i}|P(u)|}{|P(S_i)|} - 1\right).$$

This could be reduced further by extending the protocol to "test the waters" by checking with a small sample of neighbors for each list prior to broadcasting an item, but this would require the agents to be more aware of the system's structure. The original distributed algorithm is more complicated to analyze in depth. All queries prior to the discovery of a SIG item are spam, and as the propagation of those items begins many more queries will be sent to agents that do not hold items of interest. By itself, the former consists of $\ell\frac{\eta}{\min_i\{|P(S_i)|\}}$ messages, which by itself will be more spam than the mailing-list model in many domains (when $\eta \gg \mu$).

As noted earlier, we assume that SIG members have something meaningful in common, i.e., $\frac{P(u)}{\eta} < \frac{P(S)}{P(v)}$. However in the mailing-list model is is also desirable for the ratio $|P(u)|/|P(S)|$ to be $O(1)$, to prevent an inordinate number of broadcasts from being made. This is in addition to the requirements of [5], in which the amount of superfluous traffic is not a consideration.

The following network model removes the constraint on $|P(u)|/|P(S)|$ and replaces the presence of cliques with the need for a network in which subgroups of vertices are expected to be highly connected (to the tune of an large fraction λ of their members) and which is an expander graph with a small expansion coefficient.

6 The Word-of-Mouth Model

In a random graph, it is highly unlikely that SIGs will be represented by cliques. Still, using a sample-and-share approach to spreading information, we could achieve similar results to a mailing-list if SIGs were sufficiently-connected subgraphs in G. In random scale-free networks it appears that the connected portions of SIGs represent most of the members of the SIG under certain circumstances (β_G significantly less than β_0, and $|S|$ a respectable fraction of the population size $|U|$). If most of a SIG's nodes formed a connected component, either due to this result or by construction[6], it would be appropriate to have an agent A's neighbors sample items A has liked and spread those items further if they also like the item. For ease of analysis, we will assume that nodes remember what has been recommended to them, and do not resample objects or recommend objects redundantly over edges in the network.

If an item in $P(S)$ is found and propagated to $\lambda|S|$ or more SIG members, it will also cause non-SIG members to test the item – in particular those users adjacent to the SIG in the network. A SIG member finding something in $P(S)$ would lead to $(1+\gamma)|S|$ total users testing the announced item, where γ is the expansion coefficient of the graph. For "Internet-like" SFNs[7] as in [12] Gkantsidis et al. show the core of the network has expansion properties, and that the second eigenvalue λ_2 of a stochastic matrix corresponding to a random walk on the graph is bounded as

$$1 - \Omega\left(\frac{1}{\log n}\right) < \lambda_2 < 1 - \Omega\left(\frac{1}{\log^2 n}\right).$$

For large networks this means the eigenvalue gap is not large, implying γ is small [15]. As in the SIG model, false alarms are possible, and while idiosyncratic interests should not propagate very far $(1+\gamma)|S|$ can be used as a conservative estimate. Taking this result across all SIGs, the system recommendation complexity is at worst $\ell(1+\gamma)|S|\frac{|P(u)|}{|P(S)|}$, making the total sample complexity of the word-of-mouth model

$$\ell\left(\frac{\eta}{\min_i\{|P(S_i)|\}} + \mathrm{avg}_i\left\{(1+\gamma)|S_i|\frac{\mathrm{avg}_{u\in S_i}|P(u)|}{|P(S_i)|}\right\}\right).$$

This is comparable to the mailing-list model in performance, with the caveat that only some fraction of the population $\lambda|U|$ has satisfaction guaranteed and more or less spam may be generated depending on γ.

In an SFN, this algorithm will require as much communication as there are edges within the SIG and at the boundary to the subgraph of S. However, announcing personal interests will be only a fraction of that work, and will vary depending on the popularity of the actual item discovered. This hints at the presence of *equitability*; unpopular items will not be forwarded to large groups of users as they were in the mailing-list model, and popular items will satisfy more users for the system's trouble. This also suggests the theoretical result based on SIGs may be a coarse approximation of the actual algorithm's performance, because SIGs are no longer strictly defined. Further analysis is necessary.

[6] The results in Section 4 are for SFNs created directly from a degree distribution, but SFNs can be formed by processes that exhibit growth and preferential attachment [4]. One could imagine a network growing with *interest-driven* preferential attachment. Assortativity [20] may be most important side effect of this; for our purposes, biased failure modes would need to be studied in this context.

[7] In [12] these SFNs are made with a power-law degree distribution with $2 < \beta < 3$ but are altered to ensure there is a network "core" of minimum degree 3, to which all nodes of degree 1 or 2 are connected. They are fully connected, while a random SFN with the same β would almost surely have a large number of secondary components.

7 Conclusions and Future Work

By explicitly considering the role of the network and limiting the scope of communication in a distributed recommender system, the mailing-list model and the word-of-mouth model both appear to do better in terms of sampling, communication, and spam complexity than Awerbuch's original work. Although we are particularly interested in the ability of agents to share recommendations to one another on behalf of the users they represent, our work is applicable to a more general context than DRSs. It provides a formal basis for agents to share information with only their nearest neighbors under certain circumstances, with an understanding of when a large portion of the interested agents will eventually receive that information.

This is part of ongoing work in distributed systems and the use of agents that share information to enhance distributed learning. We are confirming the new SFN results in simulation, and we realize the conductance properties of SFNs and the affects of assortativity need to be more thoroughly studied. The current conductance result depends on the low conductance of an SFN when $2 < \beta < 3$, which corresponds to the parameter space in which SIGs must represent a large fraction of the entire population (these two results are related – in a graph with poor expansion properties, more bottlenecks exist that could fail and fragment the graph). In addition, for $2 < \beta < \beta_0$ we do not know of a published result on the precise fractional size of the giant component of the graph, and we are preparing to publish our experimental plot of this – it does not appear to approach 1 in any reasonable limit as it does for $\beta < 2$. This increases the necessary SIG size to accommodate λ, and increases the contrast of our work to [8].

Both network models shown would benefit if agents could distinguish their personal user's interests from those of each SIG or from those of their neighbors, eliminating "spam" to the degree such assessments were accurate. Such information could also be used to dynamically improve the graph structure in the word-of-mouth model, if highly-correlated neighbors were introduced to one another. If features are added to the products of the current model, content-based learning methods would be the next enabling step for this work. Finally, we would like to identify systems that can share certain kinds of information such that this sharing will lead independent learners to converge on a common set of parameters. This would make data points the subject of recommendations, abstracted away from the domain of the learned model.

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