

CS 580 Specification of Software Systems

Homework 04: Region Labeling Proof.

Develop a proof outline for the correctness of the region-labeling program shown below with respect to the accompanying program specification. Pay special attention to structuring the proof in a way that makes it easy to read. There is no need to formally prove a Hoare triple such as $\{P\} s \{Q\}$, but you need to explain clearly using natural language narrative why a specific property holds.

Program

```
Program Region Labeling
declare
  C : array[1..N, 1..N] of color
  L : array[1..N, 1..N] of label
always
   $\langle \forall i, j, p, q : 1 \leq i, j, p, q \leq N$ 
     $:: \text{SameRegion}((i, j), (p, q)) = |p - i| \leq 1 \wedge |q - j| \leq 1 \wedge C(i, j) = C(p, q) \rangle$ 
initially
   $\langle \forall i, j : 1 \leq i, j \leq N :: L(i, j) = (i, j) \rangle$ 
assign
   $\langle \forall i, j, p, q : 1 \leq i, j, p, q \leq N$ 
     $:: L(i, j) := \min(L(i, j), L(p, q)) \text{ if } \text{SameRegion}((i, j), (p, q)) \rangle$ 
end
```

Definitions

$\pi(x) = \langle \exists i, j : x = (i, j) :: 1 \leq i, j \leq N \rangle$	pixel
$x \rho y = \pi(x) \wedge \pi(y) \wedge \text{SameRegion}(x, y)$	regional neighbors
$\Gamma = \rho^*$	in the same region (reflexive, transitive, closure of ρ)

Specification

init leads-to post

stable post

$init \equiv \langle \forall x : \pi(x) :: L(x) = x \rangle$
 $post \equiv \langle \forall x, y : \pi(x) \wedge \pi(y) :: L(x) = L(y) \Leftrightarrow x \Gamma y \rangle$
or
 $init \equiv L(x) = x$
 $post \equiv L(x) = L(y) \Leftrightarrow x \Gamma y$