Problem definition

Consider a square matrix C of size N. Each cell in the matrix stores some color. All adjacent cells having the same color form a region. A simple UNITY program to label each region uniquely and a formal spec. of correctness is to be developed.

(1) Develop a simple UNITY program

a simple program is developed that generates unique labels for each region. A matrix L should store the region label assigned to each of the corresponding cells in C. To keep the program proof simple, L is updated one at a time, i.e., no multiple synchronous updates are used.

Solution overview

The idea is to pull label from neighbors with the same color. And if neighbors are not labeled yet, one of the statements will label it with globalMax +1. And if neighbors of same color are labeled, update its label to the max of neighbors of same color. The overall effect is that cells in the same region may initially be assigned different label but as the computation progresses cells belonging to same region will be updated to the max among those in the same region.

UNITY program

```
Program regionLabeling
declare
   C, L : array[1..N, 1..N] of integers
initially
 < [] p : (1,1) \le p \le (N,N) :: L[p] = 0 >
always
      < p : (1,1) <= p <= (N,N) ::
       sameRegionNeighborMax(p) = < max i : (-1, -1) <= i <= (1, 1)</pre>
                                             ^ C[p+i] = C[p]
                                   :: L[p+i] >
    [] globalMax() = < max i : (1, 1) <= i <= (N, N) :: L[i] >
assign
  < [] p : (1,1) <= p <= (N,N) ::
       L[p] = sameRegionNeighborMax(p) if sameRegionNeighborMax(p) != 0
    [] L[p] = globalMax() + 1 if sameRegionNeighborMax(p) = 0
 >
end
```

(2) Formal specification of correctness

a formal specification of the correctness of the program is written following the general form: a. init -> Post b. stable Post

```
Definition 1.
a relation, N, between two cells is defined as:
x N y : x and y are neighbors cells with same color
Spec. of correctness:
x N y ensures L[x] = L[X] ^
x !N y ensures L[x] != L[X]
```