CS580
Homework 04:
Region Labeling Proof.
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Problem definition

A proof outline for the correctness of the region-labeling program shown below with respect to the accompanying program specification is to be developed with special attention to structuring the proof in an easy to read way and with no need to formally prove a Hoare triple such as {P} s {Q} but need to explain clearly using natural language narrative why a specific property holds.

Definitions

```
π(x) = < ∃  i,j : x=(i,j) :: 1≤i,j≤N >  pixel
x ρ y = π(x) ^ π(y) ^ SameRegion(x,y)   regional neighbors
Γ = ρ*                                  in the same region
                                        (reflexive, transitive, closure of r)
```

Specification

```
init leads-to post
stable post

init ≡ L(x) = x
post ≡ L(x) = L(y) ⇔ x Γ y
```

UNITY program

```
Program Region Labeling
declare
    C : array[1..N, 1..N] of color
    L : array[1..N, 1..N] of label
always
    <[]i,j,p,q : 1≤i,j,p,q≤N
    :: SameRegion( (i,j),(p,q) ) = |p-i|≤1 ^ |q-j|≤1 ^ C(i,j)=C(p,q) >
initially
    <[]i,j : 1≤i,j≤N :: L(i,j) = (i,j) >
assign
    <[]i,j,p,q : 1≤i,j,p,q≤N
           :: L(i,j) := min(L(i,j),L(p,q)) if SameRegion( (i,j),(p,q) ) >
end
```

Proof outline

1. stable post

```
assume post "L(x) = L(y) ⇔ x Γ y" is true

this iif relationship means

"L(x) = L(y)" => x Γ y ...(1) and
x Γ y => "L(x) = L(y)" ...(2)

under this condition, looking at the statements:

L(i,j) := min(L(i,j),L(p,q)) if SameRegion( (i,j),(p,q) )

the assignment only take place when SameRegion( (i,j),(p,q) ) is true,
and from (2), when SameRegion(x, y) holds, L(x) = L(y),
therefore min(L(x),L(x)) = L(x) = L(y)
therefore L(x) = L(x) = L(y),
which means x's label doesn't change if post is assumed true.
```

2. init leads-to post

```
 From "initially" section of the UNITY program,
"init ≡ L(x) = x" holds.

select the well-founded metric to be:
N = the number of points in the same region
with label greater than the minimum of the region.

Such metric is well-founded because its minimum value is 0.

And N decreases which can be proved from
{N=k} ensures {N<k}
since the program has one statement that update label of
a point to the minimum of the region.
```