Towards practical biomolecular computers using microfluidic deoxyribozyme logic gate networks

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Abstract

We propose a new way of implementing a biomolecular computer in the laboratory, using deoxyribozyme logic gates inside a microfluidic reaction chamber. We build upon our previous work, which simulated the operation of a deoxyribozyme-based flip-flop and oscillator in a continuous stirred-tank reactor (CSTR); unfortunately, using these logic gates in a laboratory-size CSTR is too expensive because the reagent volume is too large. For a realistic microfluidic design, the properties of microfluidic flow and mixing have to be taken into account. We describe the differences between a macrofluidic system such as the CSTR and the microfluidic setting. Liquid in a microfluidic setting exhibits laminar flow, and is more difficult to mix than in the CSTR. We use a rotary mixer, and examine how it operates so that we may properly model it. We discuss the details of our mixer simulation, including our diffusion model. We discuss why having discrete phases of influx/efflux ("charging") and mixing is a necessary, and how it changes the kinetics of the system. We then show the result of simulating both a flip-flop and an oscillator inside our rotary mixing chamber, and discuss the differences in results from the CSTR simulation.

1 Introduction

Deoxyribozymes (nucleic acid enzymes) may be used as logic gates, which transform input signals, denoted by a high concentration of substrate molecules, into

output signals, which are represented by product created when the deoxyribozyme gate cleaves a substrate molecule [SMS02]. In this fashion, molecular devices have been created in the laboratory that function as a half-adder [SS03a] and a tic-tac-toe automaton [SS03b]. Furthermore, first experiments have demonstrated the linking of the output of certain deoxyribozyme gates to the input of others, which opens the prospect of creating complex logic [SSK+05].

However, these gates have so far only been used in the laboratory in very small quantities, and, quite significantly, only in closed reactors. This is due to the expense involved in purchasing large amounts of gate molecules and the substrates that act as their input. Performing experiments with these gates in closed reactor systems has the major drawback of limiting their capability to one-shot computation. Previously, we have simulated multiple gate operation in an open, continuous-influx stirred tank reactor (CSTR), and have shown the design of a flip-flop in this setting [MSMS04] as well as an oscillator. Unfortunately, no such open reactor experiment has been performed, owing to the attendant costs.

We propose a microfluidic system whereby these open reactor experiments may actually be performed in the laboratory at a modest cost in materials and apparatus. We analyze and simulate a molecular flip-flop and oscillator in a microfluidic setting. The reaction kinetics of the flip-flop and oscillator in the CSTR have already been examined in detail. Our simulation changes these kinetics by making the influx and homogeneity of the system time-dependent, varying according to our simulation of a microfluidic mixer, which doubles as the reaction chamber.

The extremely small volume of a microfluidic reaction chamber (ours is 7.54 nL) compared to a CSTR (50 mL or more) means that the same or even substantially greater concentrations of oligonucleotide gates and substrates can be obtained within the chamber even with a vastly smaller amount of gate and substrate molecules. This means that the expense of an open-reactor experiment (mostly determined by the amount of substance used—and that includes the substrates, the products, and the gates) can be reduced by several orders of magnitude, and be made reasonable. In addition to reducing expense and thereby enabling real-life open-reactor experiments, our approach has numerous other unique advantages, including a vast decrease in the time needed to perform logic operations, the possibility of keeping gates inside a chamber (allowing for pre-fabricated chambers, each implementing a certain type of logic), and the ability to link reaction chambers together with externally-controlled valves to create exceedingly complex logic, where channels between chambers could even be designed to mimic capillaries connecting living cells in which computation may be taking place *in*

vivo at some point in the future. In fact, we consider our microfluidic setting to be the proving ground for deoxyribozyme logic gate circuits for medical applications.

2 The Chemical Kinetics of Deoxyribozyme Gate Networks

The four chemical components present in our reactor are inputs, gates, substrates, and products. All of these components are oligonucleotides. The gates are deoxyribozyme molecules, and under certain input conditions they are active [SMS02]. When a gate becomes active, it cleaves substrate molecules to create product molecules. In more technical terms, the enzymatic (active) gate is a phosphodiesterase: it catalyzes an oligonucleotide cleavage reaction. Input molecules can either activate or deactivate a gate. The effect that a particular type of input molecule has on a gate defines its function. For instance, a simple inverter, or NOT gate, will be active, and cleave substrate to produce product, until an input molecule binds to it, making it inactive. The concentration of product in the system is the output signal of the gate, where a high concentration of product is read as true and a low concentration is read as false (the same is true for high or low input concentrations). Thus, for the NOT gate example, the concentration of product in the system becomes high when there is no input and becomes low when input molecules are added, as the input molecules deactivate all of the gate molecules and product is no longer being cleaved from substrate. This example of the NOT gate's operation depends on its being in an open reactor, however—if it is in a closed reactor, the product concentration can never go from high to low, but in an open reactor, product is always being removed from the system as part of the system's efflux.

In order to model the operation of these logic gates, we must be well informed of their basic chemical kinetics. The kinetics of the YES gate have been thoroughly examined [MSMS04], and we use those results here. In this examination, it is assumed that the bonding between gate and input molecules is instantaneous and complete, since it is known that the cleavage and separation of the substrate molecules into product molecules is the slowest of the reactions, and thus is the rate-limiting process. The rate at which product is produced by a gate is $\frac{dP}{dt} = \beta S G_A$, where P is the product concentration, β is the reaction rate constant, S is the substrate concentration, and G_A is the concentration of active gates.

It has been empirically determined that the reaction rate constant for the YES gate is approximately $\beta = 5 \cdot 10^{-7}\,\mathrm{nM}^{-1}\mathrm{s}^{-1}$. This value will be assumed as the reaction rate for all deoxyribozyme gates mentioned herein.

The chemical kinetics of an entire system of gates, substrates, inputs, and products in an open, microfluidic reactor can be modelled with a set of coupled differential equations. An example is the case of the inverter, or NOT gate, where the set of equations is:

$$\frac{dG}{dT} = \frac{G^m(T) - E(T)G(T)}{V}$$

$$\frac{dI}{dT} = \frac{I^m(T) - E(T)I(T)}{V}$$

$$\frac{dP}{dT} = \beta H(T)S(T)\max(0, G(T) - I(T)) - \frac{E(T)P(T)}{V}$$

$$\frac{dS}{dT} = \frac{S^m(T)}{V} - \beta H(T)S(T)\max(0, G(T) - I(T)) - \frac{E(T)S(T)}{V}$$

where I^m, G^m, and S^m are the rates of molar influx of the respective chemical species, V is the volume of the reactor, $E(T)$ is the rate of volume efflux, β is the reaction rate constant, and $H(T)$ is a number representing the volume fraction of the reaction chamber that is homogeneous at time T. The influx and efflux of the reactor are time-dependent, because the reactor must close off its input and output periodically in order to mix its contents (*vide infra*). The variable $H(T)$ is needed because in a microfluidic system we cannot assume that the contents of the reactor are always perfectly mixed. New substrate that comes into the system during the period of influx must be mixed before it may react with the gates in the system. This allows for separate influx streams for new gates and for substrates and input molecules. It also allows for the possibility that new gates never enter or leave the system at all; instead, they could be attached to beads which cannot escape semi-permeable membranes at the entrances and exits to the chamber. In either case, only the portion of the total substrate in the chamber that has been mixed with the solution containing the gates may react. The specifics of how the efflux and the homogeneity of the system are calculated are discussed in the next section.

3 Microfluidics

In order to simulate an open microfluidic reaction system, we must first analyze the properties of such a system. First, and most obviously, the size of a microfluidic reaction chamber is dramatically small compared to the size of a more conventional open reaction chamber, such as a CSTR. The volume of the smallest CSTR that can be readily assembled is on the order of 50 mL (our previous work used 500 mL), while the volume of a microfluidic reaction chamber is often on the order of 5 nL—a difference of seven orders of magnitude. The reaction chamber we chose for our simulation has a volume of 7.54 nL. This very small volume allows us to have very high concentrations of gate, substrate, input, and product molecules, while keeping the actual number of molecules in the system low.

Fluid flow in microfluidic channels and reaction chambers is different from the flow in a large-scale system because of the very small volumes involved. Namely, the flow is invariably laminar, i.e., there is almost no turbulence (the Reynolds number of the flowing liquids is typically well below 100). This presents a peculiar challenge: two fluids flowing side by side in a microfluidic channel do not mix except by diffusion, which is a very slow process, but the fluid already in an open reaction chamber must mix quickly with new fluid flowing into the chamber, which contains new supplies of substrates, inputs, and gates, to allow the reaction to continue. This necessitates the use of an active microfluidic mixer for our reaction chamber, to speed up the mixing of the fluids greatly over normal mixing by diffusion.

We have chosen a microfluidic rotary pump to act as our open reaction chamber [CUQ01]. This device is an active mixer, mixing the solution within it by pumping it in a circular loop. The design of the device is shown in Figure 1. It consists of a bottom layer with fluid channels, and a top layer with pneumatic actuation channels. Both layers are fabricated with multilayer soft lithography [UCT+00]. One input channel in the bottom layer is used for substrate and input influx, while the other channel is used for gate influx—this separation is to keep the substrate and gates from reacting before they have entered the reaction chamber. The pneumatic actuation channels on the top layer form microvalves wherever they intersect with the fluid channels on the bottom layer. A valve is closed when the air channel is pressurized and open when it is not. The actual reaction chamber is the central loop in the diagram. Actuating the valves around the perimeter of the loop in a certain sequence peristaltically pumps the fluid inside either clockwise or counterclockwise. The frequency of actuation controls the speed at which the fluid rotates.

Continuous-flow mixing is possible with this reaction chamber, but it is not feasible for our purposes for two reasons. The first is that the mixer does not completely mix objects with relatively high diffusion constants, such as very large molecules and 1 μm beads, when the flow is continuous. Even if sufficient mixing of oligonucleotide molecules of the size we currently use could be achieved by using a low flow rate or widening and lengthening the mixer loop, this is not conducive to the possibility of attaching gates to beads, so that they may be kept always in the chamber by using semi-permeable membranes. The second problem is that the flow rate required for continuous-flow operation would have to be unreasonably low, in order to allow the gates involved to produce product molecules faster than they are removed from the system. Therefore, our model of the rotary mixing chamber uses two discrete, alternating phases: an influx and efflux, or "charging" phase, during which the valves at the chamber entrance and exit are open and the rotary pump is not operating, and a mixing phase, during which the valves at the entrance and exit of the chamber are closed and the pump is operating.

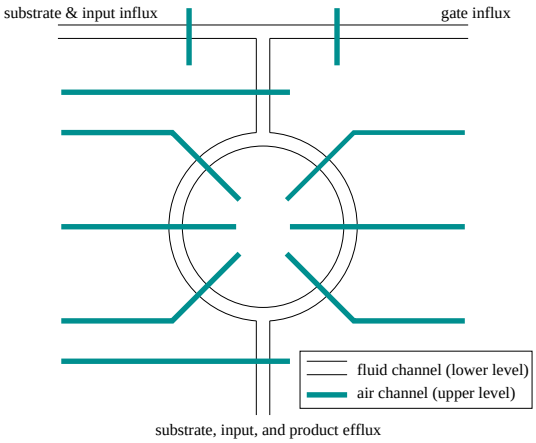

Figure 1: The rotary mixer. The air channels form microvalves wherever they intersect with the fluid channels.

4 Mixing and Diffusion

Through a combination of factors, the rotary pumping in the mixing chamber greatly increases the mixing speed over spontaneous diffusion. The parabolic flow

profile present in the tiny microfluidic channels (the fluid in the middle moves much faster than the fluid on the very edge, which is stationary) causes interface elongation, which, combined with the shallow channel depth, causes the mixing substances to fold around one another [CUQ01]. Where once the two fluids being mixed were completely separated, one in one half of the chamber and the other in the other half, after sufficient mixing time the width of the channel holds many alternating sections ("folds") of the two fluids.

We can think of a substance as being completely homogeneous in the chamber when enough of that substance has diffused, from the fluid it was in originally, across a characteristic distance l, which is the farthest the substance must penetrate into the second fluid. Initially, we have $l_0 = r_0$, where r_0 is half the width of the channel that forms the mixing chamber. This is because we can assume that initially, when there is perhaps one fold in the chamber, the two liquids are side by side, with one liquid filling up half of the channel and the other filling up the other half. In order for a substance to be completely mixed in this situation, it must diffuse from its liquid all the way across half the width of the channel, until it reaches the far edge of the second solution at the chamber's wall. As the mixer continues running, however, the characteristic distance over which the fluids must diffuse to mix is reduced proportionally to the number of rotations, because of the liquids' folding around each other. Specifically, we have $l = l_0/kt$, where k is a constant coefficient determined by the total length of the loop and the pumping speed [CUQ01].

Knowing how the maximum characteristic diffusion distance changes over time, it is possible to model the mixing of the system using a diffusion equation. We use an equation which models diffusion of a substance in a fluid that is extended in all dimensions, where the substance is initially confined in one dimension in the region $-h < x < +h$. The regions from $-h$ to $-\infty$ and from $+h$ to $+\infty$ contain fluid with zero initial concentration of the diffusing substance. Substance is free to diffuse in either direction—solutions may be found for negative and positive values of x. The equation is:

$$C(x,t) = \frac{1}{2}C_0\left\{\operatorname{erf}\frac{x-h}{2\sqrt{Dt}} + \operatorname{erf}\frac{x+h}{2\sqrt{Dt}}\right\}$$

where $C(x,t)$ is the concentration of the diffusing substance at location x and time t, C_0 is the concentration initially within the region $-h < x < +h$, D is the diffusion constant of the diffusing substance, and erf is the standard mathematical error function ($\operatorname{erf} z = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-\eta^2)d\eta$) [Cra75]. Because the liquids are folding

around each other, both h, which bounds the fluid the substance must diffuse out of, and the farthest distance $x = h + l$ to which it must diffuse, are time-dependent. We already know that $l = l_0/kt$, and, since we shall assume that the two fluids have equal-size folds at any given time t, we know that $h = l$.

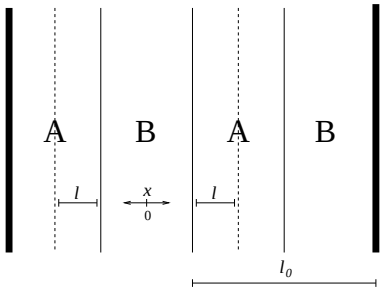

Figure 2: Folds in a section of the mixer channel.

The only problem with using these equations to model our rotary mixer is that we do not know what the constant k is in the equation for the length of diffusion l. We do know, however, from empirical evidence [CUQ01], that at a certain pumping speed it takes 30 seconds to completely mix a solution containing dye with a solution containing $1\mu m$ beads. We can use this fact to estimate k by noting the value of k for which the concentration of diffusing beads at the maximum mixing distance l is approximately equal to the concentration of beads in the middle of the fluid containing them originally (at $x = 0$) at time $t = 30\,\mathrm{s}$. Conservatively, we choose to focus on the beads for determining when the fluids are completely mixed because they have a diffusion constant that is much lower than the dye, and thus they diffuse much more slowly. The diffusion constant of the beads is $D = 2.5 \cdot 10^{-9}\,\mathrm{cm^2 s^{-1}}$. We find that the concentrations are 97.72% the same when $k = 2$. We do not attempt to get the concentrations to be 100% equivalent, because we realize that the diffusion equation becomes less accurate at the boundary condition at the end of the mixing process, since it assumes that the fluid extends infinitely and substance does not diffuse completely during the duration of the experiment. Also, it is much safer for our purposes to underestimate k than overestimate it, as an underestimate leads to slower mixing, which has the potential to disrupt the kinetics of our chemical system. We shall see, however, that it does not disrupt it enough to cause the logic that the gates perform to break down.

Using our value of $k = 2$, and the equations for the characteristic length of diffusion and the concentration of a diffusing substance at time t and position x, we

can simulate the mixing chamber. There are no beads involved in our experiments; rather, we are only mixing fluids with gate, substrate, and input molecules. So, in accordance with the length of our oligonucleotide strands, we use the diffusion constant for a DNA 50-mer, which is $1.8 \cdot 10^{-7}\,\mathrm{cm}^2\mathrm{s}^{-1}$, in our mixing simulation. The mixing affects the differential equations describing the kinetics of the chemical system within the chamber by way of $H(T)$, which is a function of time (see Section 2). This function returns the fraction of the reaction chamber which is mixed. As noted earlier, during an experiment the rotary mixer alternates spending time in a charging phase, where there is an influx of new substrate, input, and gate molecules and an efflux of homogeneous solution, and a mixing phase, where the influx and efflux valves are closed and the rotary pump is turned on.

5 A Flip-Flop

Now that we can model the microfluidic mixing chamber, we must implement interesting logic in it using networks of deoxyribozyme-based logic gates. Since we are using an open system, we can create circuits which have persistent information that can be accessed and changed over time. The simplest such digital circuit is the *flip-flop*. A flip-flop is a bistable system which represents a single bit of memory. It can be commanded to *set* or *reset* this bit, which causes it to enter its high or low stable state, respectively, or to simply store, or *hold*, the bit in memory, in which case it stays in the state that it was last set or reset to.

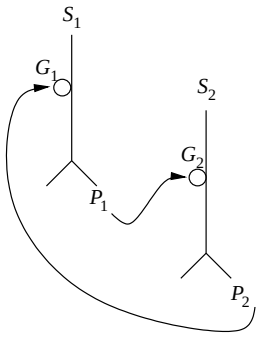

Figure 3: The flip-flop reaction network.

We simulated the operation of a biochemical flip-flop within our modeled microfluidic mixing chamber. The flip-flop was implemented as a network, shown

in Figure 3, of two deoxyribozyme-based NOT gates connected in a cycle of inhibition [MSMS04]. In this system there is no influx of input molecules, only of substrate molecules. We use the substrate molecules themselves to control the behavior of the flip-flop. A high concentration of substrate S_2 signifies a set command, a high concentration of substrate S_1 signifies a reset command, and a high concentration of both substrates is used as the hold command. The first gate, G_1, can only cleave substrate S_1, and produces product P_1. The product P_1, in turn, acts as the input molecule for the second NOT gate, G_2, inhibiting its operation. When there is little or no P_1, the second gate G_2 is active, and it cleaves substrate S_2 to produce product P_2, which acts to inhibit the operation of the first gate, G_1. We measure output from the flip-flop in terms of the concentration of the cleaved product P_2, with high or low concentrations corresponding to a logical one or zero, respectively. It is apparent that the legal commands of set, reset, and hold we mentioned earlier will perform correctly with this inhibition cycle, with certain parameters. If only substrate S_1 is present in the system, only product P_1 and no P_2 will be produced—this corresponds to the reset command. If only S_2 is in the system, only product P_2 will be produced—this corresponds to the set command. However, if both S_1 and S_2 are in the system, we will stay at whatever state we were at previously, because whichever gate was originally producing more product than the other will inhibit the operation of the other gate, and will itself become less inhibited as a result, and thus eventually will become the only operating gate—this corresponds to the hold state. This operation requires that the concentrations of the gates are equal, for symmetry, and also that the efflux of the system is not greater than the rate at which the gates can produce product, so product is not being removed faster than it is being created.

The details of this bistable flip-flop system in a CSTR were examined thoroughly in previous work [MSMS04]. In the case of implementing this gate network in a microfluidic rotary mixer, we first define $S_1^m(T)$ and $S_2^m(T)$ to be the variable *molecular* influx of the substrates at time T, with which the flip-flop is controlled. The variable molecular influx of gate molecules, which enter the reactor in a separate stream from the substrate and input molecules, is given by $G_1^m(T)$ and $G_2^m(T)$. The rate of efflux is given by $E(T)$, and is time-dependent, because the system only has influx and efflux during its charging phase, and not during its mixing phase. We define $G_1(T)$, $G_2(T)$, $P_1(T)$, $P_2(T)$, $S_1(T)$, and $S_2(T)$ to be the concentrations within the reactor at time T of gate 1, gate 2, product 1, product 2, substrate 1, and substrate 2, respectively. We can now represent the dynamics of the flip-flop system with a set of six coupled differential equations:

$$\frac{dG_1}{dT} = \frac{G_1^m(T) - E(T)G_1(T)}{V}$$

$$\frac{dG_2}{dT} = \frac{G_2^m(T) - E(T)G_2(T)}{V}$$

$$\frac{dP_1}{dT} = \beta_1 H(T) S_1(T) \max(0, G_1(T) - P_2(T)) - \frac{E(T)P_1(T)}{V}$$

$$\frac{dP_2}{dT} = \beta_2 H(T) S_2(T) \max(0, G_2(T) - P_1(T)) - \frac{E(T)P_2(T)}{V}$$

$$\frac{dS_1}{dT} = \frac{S_1^m(T)}{V} - \beta_1 H(T) S_1(T) \max(0, G_1(T) - P_2(T)) - \frac{E(T)S_1(T)}{V}$$

$$\frac{dS_2}{dT} = \frac{S_2^m(T)}{V} - \beta_2 H(T) S_2(T) \max(0, G_2(T) - P_1(T)) - \frac{E(T)S_2(T)}{V}$$

where β_1 and β_2 are the reaction rate constants, V is the volume of the reactor, and $H(T)$ is the fraction of the substrate molecules in the chamber which have been mixed (these are the only ones available to react).

In order to achieve flip-flop behavior with this system, we must find appropriate values for the system's efflux, the mixing rate, and the time spent by the system in its mixing phase and charging phase. We fix our mixer's high efflux at $0.12\,\mathrm{nL\,s^{-1}}$. During the charging phase, the mixer has this high efflux value, while during the mixing phase, the efflux is 0. The influx of the mixer is the same as the efflux, to maintain constant volume. We fix the mixing rate based on our empirically determined value for the constant k, which directly controls the mixing speed by determining the number of folds the mixer produces in a given amount of time. This value could be significantly adjusted in reality, as k simply depends on the length of the mixing channel and the speed of the pumping; our value of $k = 2$ reflects what we have determined to be one realistic value. With the efflux and mixing rate fixed, the only variable affecting the operation of the flip-flop is the time the mixing chamber spends in its charging and mixing phases. We find empirically that it works very well to spend 15 seconds in the charging phase and 15 seconds in the mixing phase.

With these parameters, Figure 4 shows the system of equations numerically integrated over a period of $1.2 \cdot 10^4\,\mathrm{s}$. The concentration of each type of gate molecule in the chamber was held steady at 130 nM, with the molecular influx of gates always matching the efflux of gates. We move the system from set, to hold, to reset at $2.5 \cdot 10^3\,\mathrm{s}$ intervals. The rapid, shallow oscillations in product concentration are due to the alternating, discrete sections of charging and mixing the system experiences.

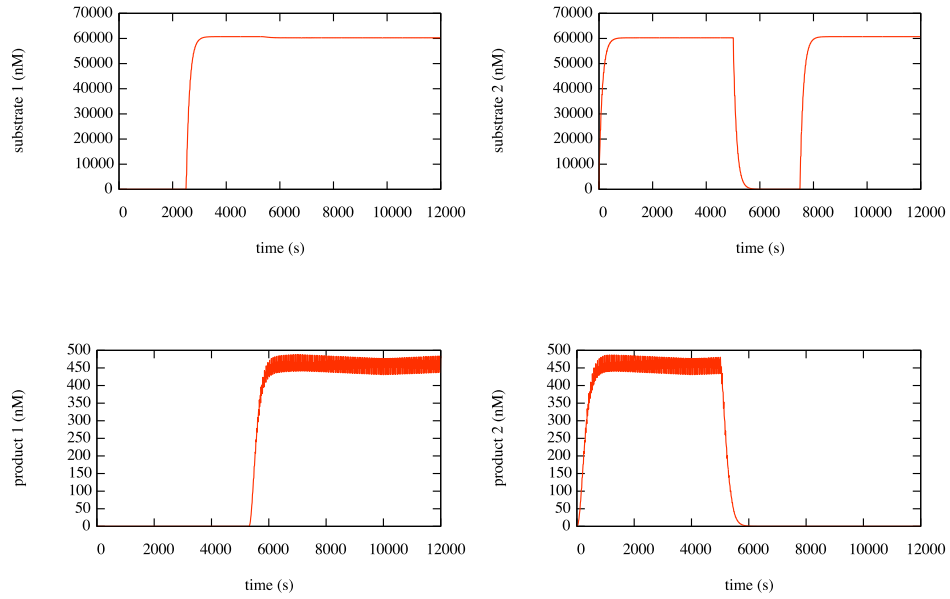

Figure 4: The flip-flop moved from set, to hold, to reset commands at 2500 s intervals.

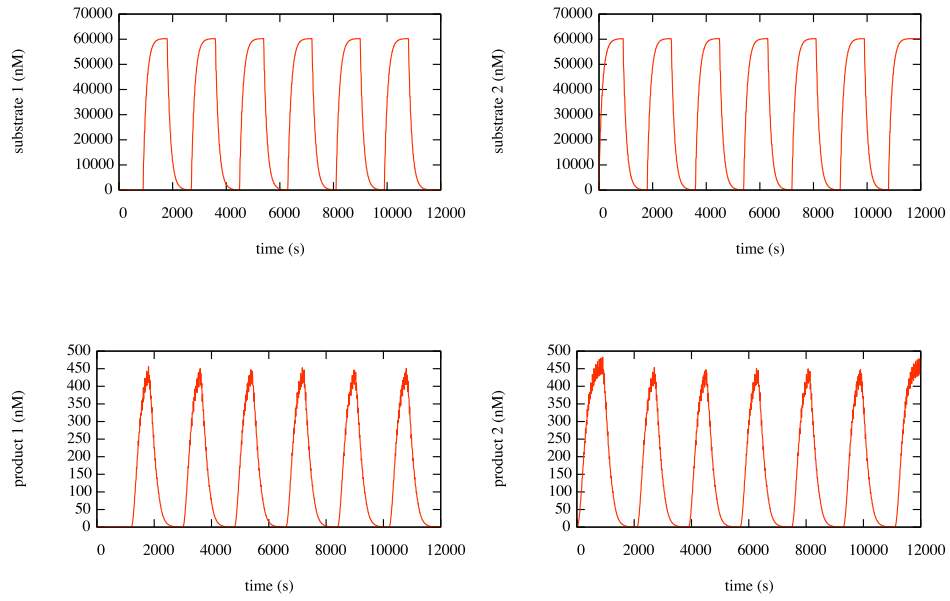

Figure 5: The flip-flop operating at its maximum switching speed.

Figure 5 shows the flip-flop switching between the set and reset commands at its maximum rate of speed. This rate was empirically determined to be about 900 seconds given to each command. This is over 65 times faster than simulations showed the flip-flop's maximum switching rate to be in the CSTR. We should also note that the concentration of substrate within the reaction chamber is a factor of 10 higher than in the CSTR simulation. Because the volume of our mixing chamber is over 7 orders of magnitude smaller than the volume of the CSTR, however, and our flow rate is 5 orders of magnitude lower, the total number of moles of substrate used in the microfluidic simulation is vastly lower than in the CSTR simulation. In fact, the molecular influx of a high substrate signal is only about $7.29\,\mathrm{fmol\,s^{-1}}$. Thus, in the span of a 1.2×10^4 s experiment (a little over three hours), less than two tenths of a nanomole of substrate is used.

6 An Oscillator

If we increase the number of enzymatic NOT gates in our microfluidic reaction chamber to any odd number greater than one, we can create a biochemical oscillator. We will focus on a network of three NOT gates for simplicity. The three gates are, as before, connected in a cycle of inhibition. We require three different substrates, one matching each gate. Each gate cleaves its substrate into a unique product which inhibits one other gate. Gate G_1 cleaves substrate S_1 to produce product P_1, which acts as input to gate G_2, inhibiting it, while gate G_2 cleaves S_2 to produce P_2, which inhibits gate G_3, and finally gate G_3 cleaves the substrate S_3 to produce P_3, which inhibits gate G_1. As before, there will be one input stream which is a mixed solution containing the three types of substrate molecules, and another stream containing fresh gate molecules. The output of the system will be a solution containing only substrate and product molecules.

We define $G_1(T)$, $G_2(T)$, $G_3(T)$, $S_1(T)$, $S_2(T)$, $S_3(T)$, $P_1(T)$, $P_2(T)$, and $P_3(T)$ to be the concentrations within the reactor at time T of the gates, substrates, and products. We define $G_1^m(T)$, $G_2^m(T)$, $G_3^m(T)$, $S_1^m(T)$, $S_2^m(T)$, and $S_3^m(T)$ to be the molecular influx rate of each species which is replenished during the charging phase. We may describe the system dynamics with the following nine coupled differential equations:

$$\frac{dG_1}{dT} = \frac{G_1^m(T) - E(T)G_1(T)}{V}$$

$$\frac{dG_2}{dT} = \frac{G_2^m(T) - E(T)G_2(T)}{V}$$

$$\frac{dG_3}{dT} = \frac{G_3^m(T) - E(T)G_3(T)}{V}$$

$$\frac{dP_1}{dT} = \beta_1 H(T) S_1(T) \max(0, G_1(T) - P_3(T)) - \frac{E(T)P_1(T)}{V}$$

$$\frac{dP_2}{dT} = \beta_2 H(T) S_2(T) \max(0, G_2(T) - P_1(T)) - \frac{E(T)P_2(T)}{V}$$

$$\frac{dP_3}{dT} = \beta_3 H(T) S_3(T) \max(0, G_3(T) - P_2(T)) - \frac{E(T)P_3(T)}{V}$$

$$\frac{dS_1}{dT} = \frac{S_1^m(T)}{V} - \beta_1 H(T) S_1(T) \max(0, G_1(T) - P_3(T)) - \frac{E(T)S_1(T)}{V}$$

$$\frac{dS_2}{dT} = \frac{S_2^m(T)}{V} - \beta_2 H(T) S_2(T) \max(0, G_2(T) - P_1(T)) - \frac{E(T)S_2(T)}{V}$$

$$\frac{dS_3}{dT} = \frac{S_3^m(T)}{V} - \beta_3 H(T) S_3(T) \max(0, G_3(T) - P_2(T)) - \frac{E(T)S_3(T)}{V}$$

where β_1, β_2, and β_3 are the reaction rate constants, V is the volume of the reactor, $E(T)$ is the time-dependent volumetric efflux, and $H(T)$ is the fraction of the reaction chamber which is homogeneous at time T.

The conditions under which the oscillator will oscillate in a CSTR have been examined previously [MSMS04]. To simplify things, this examination assumed that the concentration of substrate molecules in the chamber was constant, because, although these concentrations do oscillate, they are always much higher than the oscillating concentrations of the products. Using this assumption, linear approximations can be made to explicitly solve the differential equations for the oscillating product concentrations. These approximations give us a way to specify the center and period of the oscillations by setting an appropriate influx of substrate molecules and an appropriate concentration of gates. Our circumstances differ from the CSTR in that the efflux alternates between off and on, and the system is almost never completely homogeneous. We recognize that never less than 76% of the system is homogeneous at any given time, however, and so it is reasonable to assume constant, complete homogeneity, and constant efflux, in order to use the approximations from the CSTR research as a starting point for specifying the period and center of our oscillator.

We set our efflux rate for the charging cycle equal to the rate we used for our flip-flop experiment, $0.12\,\mathrm{nL\,s^{-1}}$. We use the same period of 15 seconds in the charging phase and 15 seconds in the mixing phase that worked well for the flip-flop simulation. Based on the efflux rate, we use the linear approximations derived from the CSTR simulation research to calculate an estimate for the gate concentration and substrate influx needed for oscillations of period 250 seconds, centered at $1\,\mu$M. We find we should keep each of the gate concentrations steady at 1500 nM, while the molecular influx for each substrate should be set to $7.29 \times 10^{-6}\,\mathrm{nM\,s^{-1}}$. Figure 6 shows the results of integration over a 5000 second period with these initial values. We can see that the actual period is 480 seconds, and the actual center is close to $1.5\,\mu$M. The linear approximations were off by about 20% in the CSTR simulation; in our simulation, the period estimation is just over half the actual period, and the center estimation is off by about 50%. There are two reasons for this. One is the fact that we assumed our efflux rate and reactor homogeneity to be constant in order to use the same approximations that worked in the CSTR setting. Another, more instrumental reason stems from the fact that reactions happen much more quickly in our microfluidic system, since we have a much higher concentration of reagents. This causes the nonlinear terms that are not taken into account in the linear approximations to become much more prominent. More analysis is required to find a more accurate way to specify the

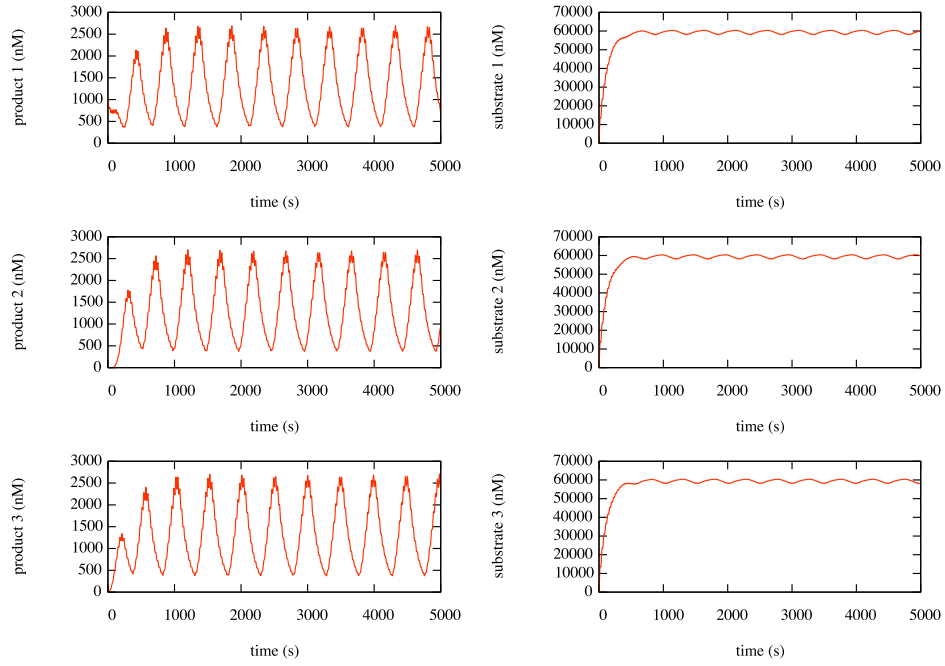

Figure 6: The oscillator system operating with a period of 480 s and a center of 1.5 μM.

period and center of our oscillations.

7 Related Work

Microfluidics has previously been proposed as a laboratory implementation technique for automating DNA computation algorithms [LL03, GR99, LMRPS]. It is even possible to use microfluidics for computational purposes as a purely mechanical substrate, i.e., without chemical reactions [CPW$^+$01, FDK$^+$03, VMM04]. That fluidics can be used thus has been known for a long time [FP70], but microfluidics for the first time offers the potential for building relatively complex devices [TMQ02, GEQ03, HQ03].

Microfluidic mixing is a difficult problem. While we have opted for the rotary mixing chamber design as one for which modelling the kinetics of mixing is within reach, other designs have been proposed; droplet-based mixing [PPF03,

TSLI03, STI03] is especially attractive [GSI04]. Analysis of mixing remains a challenging problem [Wig03, SM03]. Related to mixing, or achieving uniform concentration, is the problem of achieving particular spatiotemporally nonuniform concentrations [JDC+00, DCJW01, JBD+02].

8 Conclusions

Networks of deoxyribozyme-based logic gates can operate in a microfluidic environment. This is the first feasible setting in which open-reactor experiments using these gates may be conducted in the laboratory. The immediate and obvious advantage of this approach, compared to using a larger open reactor, is a massive savings of cost and time. Our simulations of a flip-flop and an oscillator in such a setting show that useful microfluidic experiments could be conducted in mere hours, rather than the days or weeks it would take to see results in a large, continuous-flow stirred tank reactor. Perhaps most astonishingly, the extremely small volume of a microfluidic reactor means that a three-hour experiment could cost less than $50 in reagents, even though deoxyribozyme-based gates and the oligonucleotide substrates and inputs which they react with can cost as much as $40 per nanomole.

Our microfluidic reaction chambers are also very conducive to being networked together, with control logic outside the system operating valves on the channels connecting them. We can investigate the possibility of attaching gate molecules to beads, and keeping them within a chamber by placing semi-permeable membranes at the chamber entrances and exits. With such a system, we could keep discrete sections of logic separate from each other when desired, and redirect outputs and inputs selectively. This may be especially useful if certain types of gates whose logic we wish to connect actually conflict undesirably with each other if they are placed in the same chamber (by partially binding to each others' input or substrate molecules, for example). We believe that using microfluidic rotary mixing chambers to implement complex logic with deoxyribozyme-based gates in actual laboratory experiments is the first step toward completely understanding their potential, and eventually even deploying them in situations as complex as living cells.

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