Give definitions for the following functions in Haskell.

1. The function \textit{stutter} takes a list of elements and returns a list where every element has been duplicated. For example,

   *Main> stutter "Hello World"
   "HHeelllloo WWoorrarllldd"
   *Main> stutter [1,2,3]
   [1,1,2,2,3,3]

2. The function \textit{compress} eliminates consecutive duplicate elements of a list. For example,

   *Main> compress "HHeelllloo WWoorrarllldd"
   "Helo World"
   *Main> compress [1,2,2,3,3,3]
   [1,2,3]

3. The function \textit{findIndices} takes a predicate and a list as arguments and returns a list of numbers indicating the positions of elements in the list which satisfy the predicate. For example,

   *Main> findIndices (< 'a') "AbCdef"
   [0,2]
   *Main> findIndices (== 0) [1,2,0,3,0]
   [2,4]

   The function \textit{intersect} takes two lists as arguments and returns a list of elements common to both lists. For example

   *Main> intersect "abc" "cat"
   "ac"
   *Main> intersect [1,2,3] [8]
   []
   *Main> intersect [3,2,1] [1,2,3]
   [3,2,1]

4. The function \textit{isPrefixOf} takes two lists as argument and returns \textit{True} iff the first list is a prefix of the second list. For example,

   *Main> "foo" `isPrefixOf` "foobar"
   True
   *Main> isPrefixOf [1,2,3] [4,5,6]
   False
5. The function `isSuffixOf` takes two lists as argument and returns `True` iff the first list is a suffix of the second list. For example,

*Main> "bar" `isSuffixOf` "foobar"
True
*Main> isSuffixOf [1,2,3] [4,5,6]
False

6. The dot product of two vectors \(\vec{u}\) and \(\vec{v}\) of length \(n\) (written \(\vec{u} \cdot \vec{v}\)) is defined to be \(\sum_{i=1}^{n} u_i v_i\). Define a function `dot` which takes two lists of numbers of equal length and returns their dot product.

*Main> [0,0,1] `dot` [0,1,0]
0

7. The function `increasing` takes a list of enumerable elements as its argument and returns `True` if the list is sorted in increasing order and `False` otherwise.

*Main> increasing "ABCD"
True
*Main> increasing [100,99..1]
False

Write `increasing`.

8. To ‘decimate’ literally means to kill every tenth man (it was a punishment in the Roman legions). Define a function `decimate` which removes every tenth element from a list. For example,

*Main> decimate [1..21]
[1,2,3,4,5,6,7,8,9,11,12,13,14,15,16,17,18,19,21]

9. Define a function `encipher` which takes two lists of equal length and a third list. It uses the first two lists to define a substitution cipher which it uses to encipher the third list. For example,

*Main> encipher ['A'..'Z'] ['a'..'z'] "THIS" "this"

10. Define a function `prefixSum` which takes a list of numbers as its argument and returns a list of sums of all prefixes of the list. For example,
11. The function \textit{select} takes a predicate and two lists as arguments and returns a list composed of elements from the second list in those positions where the predicate, when applied to the element in the corresponding positions of the first list, returns \textit{True}.

\begin{verbatim}
*Main> prefixSum [1..10]
[1,3,6,10,15,21,28,36,45,55]
*Main> prefixSum [2, 5]
[2, 7]
\end{verbatim}

\begin{verbatim}
11. The function \textit{select} takes a predicate and two lists as arguments and returns a list composed of elements from the second list in those positions where the predicate, when applied to the element in the corresponding positions of the first list, returns \textit{True}.

*Main> :t select
select :: (t -> Bool) -> [t] -> [a] -> [a]

*Main> select even [1..26] "abcdefghijklmnopqrstuvwxyz"
"bdfhjlnprtvxz"
*Main> select (<= 'g') "abcdefghijklmnopqrstuvwxyz" [1..26]
[1,2,3,4,5,6,7]
\end{verbatim}

12. The function \textit{numbers} which takes a list of integers as its argument and returns the integer which has those numbers as digits. For example,

\begin{verbatim}
*Main> numbers [1..4]
1234

Write \textit{numbers} using a tail-recursive helper function defined inside of a \textit{let} expression or using \textit{where}.
\end{verbatim}