

# CS 422/522: Image and Pattern Analysis

## Homework 1 (Spring '05)

### 1 Theory

1. A p.d.f. for a discrete random variable  $X$  is defined as follows:

$$f_X(x) = \begin{cases} 2x/9 & \text{if } 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of the c.d.f. at 1:

$$F_X(1) = \int_0^1 f_X(x) dx.$$

2. Let  $X$  and  $Y$  be continuous random variables where  $f_X(x) = \frac{1}{\tau} e^{-x/\tau}$  and let  $Y = X^2$ . Derive an expression for  $f_Y$ . Hint: Don't forget that the inverse is not uniquely defined.

### 2 Practice

1. Write a function *cumulative-distribution-function* which takes an image as its argument and returns the discrete cumulative distribution function (c.d.f.) for the image:

$$F(j) = \frac{255}{n m} \sum_{i=0}^j H(i)$$

where  $n$  is the number of rows,  $m$  is the number of columns, and  $H$  is the grey-level histogram. You may assume that the image contains grey-levels in the range  $[0, 255]$ . The c.d.f. should be returned as a *vector*. Compute the discrete c.d.f. for the *frog* image and for an image of your choice. Hint: Although not strictly necessary, learning the Scheme *do* macro might help you.

2. Write a function *histogram-equalize* which takes an image as its argument and returns an image which has been histogram equalized using the discrete c.d.f. as a grey-level transformation. Plot the histograms for the *frog* image and for an image of your choice before and after histogram equalization.

You should also show both images before and after histogram equalization. Hint: This is easy to do using *image-map*.

3. Write a function *inverse-cumulative-distribution-function* which takes an image as its argument and returns the discrete inverse cumulative distribution function (discrete i.c.d.f.) for the image. The value of the discrete i.c.d.f.  $F^{-1}$  at  $j$  is the minimum value  $k$  such that  $F(k) \geq j$ . You may assume that grey-levels are in the range  $[0, 255]$ . The i.c.d.f. should be returned as a Scheme *vector*.
4. Write a function *histogram-match* which takes two images as its arguments and returns an image which is the result of applying the histogram matching grey-level transformation to the first image so that its histogram is matched to that of the second image. Plot the histograms for the *frog* image and for the *cropped-rad* image after its histogram has been matched to that of the *frog* image. Show the transformed *cropped-rad* image. Repeat the above for two equal sized images of your choice. Plot the histograms and show the images before and after (for the second image) histogram matching. Hint: This is easy to do using *image-map*.