CS 422/522: Digital Image Processing
Homework 3 (Fall '12)

1. The $\int$ operator takes a function, f, as its argument and returns the anti-derivative of the function: $f \xrightarrow{\int} \int f(t)dt$. Prove that the $\int$ operator is:
   (a) Linear.
   (b) Shift-invariant.

2. Prove that $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$.

3. The impulse response function of a linear, shift-invariant system is:
$$h(t) = \frac{\sin(\pi t)}{\pi t}$$
   and its input is:
$$x(t) = \cos(4\pi t) + \cos(\pi t/2).$$
   What is its output?

4. The impulse response function of a linear, shift-invariant system is:
$$h(t) = e^{-\frac{\pi t^2}{2}}$$
   and its input is:
$$x(t) = e^{j2\pi s_0 t}.$$
   What is its output?

5. The sine Gabor function is the product of a sine and a Gaussian, $f(t) = e^{-\pi t^2} \sin(2\pi s_0 t)$. Give an expression for $F(s)$, the Fourier transform of $f(t)$.

6. The function, $f(t)$, is defined as:
$$f(t) = \begin{cases} 1 & \text{if } |at - b| \leq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$
   Give an expression for $F(s)$, the Fourier transform of $f(t)$.

7. The transfer function of a linear shift invariant system is $H(s) = 1/s$. The impulse response function, $h(t)$, is $\mathcal{F}^{-1}\{H(s)\}$. Give an expression for $g(t)$ where:

$$g(t) = \int_{-\infty}^{\infty} e^{\,j2\pi s_0 \tau} h(t-\tau)\, d\tau.$$

8. Compute the Fourier transform of $f(t) = -2\pi t\, e^{-\pi t^2} \cos(2\pi s_0 t)$. Hint: What is $\frac{d\left(e^{-\pi t^2}\right)}{dt}$?