1 Linear Shift-Invariant Systems

1. The $\int$ operator takes a function, $f$, as its argument and returns the antiderivative of the function: $f \xrightarrow{\int} \int f(t) dt$. Prove that the $\int$ operator is:

(a) Linear.
(b) Shift-invariant.

2. Prove that $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$.

3. The impulse response function of a linear, shift-invariant system is:

$$h(t) = \frac{\sin(\pi t)}{\pi t}$$

and its input is:

$$x(t) = \cos(4\pi t) + \cos(\pi t/2).$$

What is its output?

4. The impulse response function of a linear, shift-invariant system is:

$$h(t) = e^{-\pi^2 t^2}$$

and its input is:

$$x(t) = e^{j2\pi s_0 t}.$$

What is its output?

5. The sine Gabor function is the product of a sine and a Gaussian, $f(t) = e^{-\pi^2 \sin(2\pi s_0 t)}$. Give an expression for $F(s)$, the Fourier transform of $f(t)$.

6. The function, $f(t)$, is defined as:

$$f(t) = \begin{cases} 
1 & \text{if } |at - b| \leq \frac{1}{2} \\
0 & \text{otherwise.}
\end{cases}$$

Give an expression for $F(s)$, the Fourier transform of $f(t)$. 
7. The transfer function of a linear shift invariant system is $H(s) = 1/s$. The impulse response function, $h(t)$, is $\mathcal{F}^{-1}\{H(s)\}$. Give an expression for $g(t)$ where:

$$g(t) = \int_{-\infty}^{\infty} e^{j2\pi s_0 \tau} h(t - \tau) \, d\tau.$$ 

8. Compute the Fourier transform of $f(t) = -2\pi t \, e^{-\pi t^2} \cos(2\pi s_0 t)$. Hint: What is $\frac{d(e^{-\pi t^2})}{dt}$?

2 Geometric Transformation

1. Define a function which will rotate an image about its center pixel by a given angle. Test your function on an image of your choice. See Figure 1.

2. Figure 2 shows an image take by a camera pointed at a cone shaped mirror. Define a function which computes the geometric correction yielding a $360^\circ$ panorama. See Figure 3.
Figure 2: An image take by a camera pointed at a cone shaped mirror.

Figure 3: 360° panorama computed by geometric correction of image of cone shaped mirror.