CS 530: Geometric and Probabilistic Methods in Computer Science
Homework 1 (Fall '02)

1. The joint probability distribution of two discrete random variables X (rows) and Y (columns) is given below. Determine whether X and Y are statistically independent.

$$\begin{bmatrix} 1/72 & 1/36 & 5/72 \\ 1/36 & 1/18 & 5/36 \\ 1/12 & 1/6 & 5/12 \end{bmatrix}$$

2. The joint probability distribution of two discrete random variables X (rows) and Y (columns) is given below.

$$\begin{bmatrix} 0 & 5/36 & 1/3 \\ 1/12 & 1/9 & 1/18 \\ 1/36 & 1/4 & 0 \end{bmatrix}$$

   Compute and tabulate:

   (a) Marginal probability distribution, $P_X(i)$.

   (b) Marginal probability distribution, $P_Y(j)$.

   (c) Conditional probability distribution, $P_{X|Y}(i \,|\, j)$.

   (d) Conditional probability distribution, $P_{Y|X}(j \,|\, i)$.

   [Note: i is the row index and j is the column index.]

3. Let X be a continuous random variable with p.d.f.:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}.$$

   Show that $\langle X \rangle = \mu$ and $\langle (X-\mu)^2 \rangle = \sigma^2$.

4. Prove that

$$\binom{n}{r} = \binom{n}{n-r}.$$

5. Two fair dice are rolled, one of which is red and the other is green. Let W be the difference (red number less green number). Compute and plot $p_W(i)$ for $-5 \leq i \leq 5$.

6. After years of driving through a particular intersection, you know that the probability of being stopped by a red light is 0.27. You drive through the intersection twice a day, five days a week. What is the probability of being stopped by a red light five or more times in a single week?

7. A p.d.f. is defined as follows:

$$f_X(x) = \begin{cases} 2x/9 & \text{if } 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of the c.d.f. at 1:

$$F_X(1) = \int_0^1 f_X(x)dx.$$

8. Let X and Y be continuous random variables where $f_X(x) = \frac{1}{\tau}e^{-x/\tau}$ and let $Y = X^2$. Derive an expression for p_Y.

9. Prove that $\lim_{n\to\infty}(1 - \mu/n)^n = e^{-\mu}$. Hint: Use the *Binomial Formula*.

10. Prove that

$$1 - (1-p)^n = \sum_{k=1}^{n} \binom{n}{k} p^k (1-p)^{n-k}.$$

11. Write a MATLAB function which computes the $2D$ joint histogram, G_{XY}, of a pair of images, X and Y, of equal size. Test it on the red and green components of the cactus image found on the class homepage. Display the joint histogram, G_{XY}, as a grey level image. [Note: Use the online documentation to find out how to generate hardcopies of images in MATLAB (or Octave).]

12. Write a MATLAB function which, given a joint histogram, G_{XY}, returns the marginal histograms, G_X and G_Y. Using the cactus image, verify that the marginal histograms you compute are the same as those computed using the $1D$ histogram function found on the class homepage. [Note: Use the online documentation to find out how to plot and generate hardcopies of histograms in MATLAB (or Octave).]