Grammar

- $<\!symbol\!> ::= a \mid b \mid c \mid \cdots \mid z$
- $<\!lambda\!> ::=$
  (lambda($<\!symbol\!>$) $<\!sentence\!>$)
- $<\!sentence\!> ::=$
  $<\!symbol\!> \mid <\!lambda\!> \mid (<\!sentence\!> <\!sentence\!>)$

Example Derivation

$$
\begin{aligned}
<\!sentence\!> &\rightarrow (<\!sentence\!>\ <\!sentence\!>) \\
&\rightarrow (<\!lambda\!>\ <\!sentence\!>) \\
&\rightarrow (<\!lambda\!>\ <\!lambda\!>) \\
&\rightarrow ((\text{lambda}\ (<\!symbol\!>)\ <\!sentence\!>)\ \ <\!lambda\!>) \\
&\rightarrow ((\text{lambda}\ (x)\ <\!sentence\!>)\ \ <\!lambda\!>) \\
&\rightarrow ((\text{lambda}\ (x)\ <\!symbol\!>)\ \ <\!lambda\!>) \\
&\rightarrow ((\text{lambda}\ (x)\ x)\ \ <\!lambda\!>) \\
&\rightarrow ((\text{lambda}\ (x)\ x)\ (\text{lambda}\ (<\!symbol\!>)\ \ <\!sentence\!>)) \\
&\rightarrow ((\text{lambda}\ (x)\ x)\ (\text{lambda}\ (y)\ \ <\!sentence\!>)) \\
&\rightarrow ((\text{lambda}\ (x)\ x)\ (\text{lambda}\ (y)\ \ <\!symbol\!>)) \\
&\rightarrow ((\text{lambda}\ (x)\ x)\ (\text{lambda}\ (y)\ y))
\end{aligned}
$$

β Reduction

$$((\text{lambda } (x)\ E)\ M) \to E[M/x]$$

Example Reduction

$$((\text{lambda } (x)\ x)(\text{lambda } (y)\ y))$$

$$\begin{aligned} &\to\ x[(\text{lambda } (y)\ y)/x] \\ &\to\ (\text{lambda } (y)\ y) \end{aligned}$$

Substitution Subtleties

```
((lambda (x)
   (lambda (y) (x y)))
 (y w))
```

is not

```
(lambda (y) ((y w) y))
```

It should be:

```
(lambda (v) ((y w) v))
```

Substitution Subtleties (contd.)

```
((lambda (x)
   ((lambda (x) x) x))
 (lambda (y) y))
```

is not

```
((lambda (x) (lambda (y) y))
 (lambda (y) y))
```

It should be:

```
((lambda (x) x)
 (lambda (y) y))
```

Free Variables

x is *free* in E iff any of the following is true:

- E is x.
- E is of the form (lambda (y) E') and x is free in E'.
- E is of the form (F G) and x is free in F or x is free in G.

Substitution

$$E[M/x]$$

- E is x:

$$M$$

- E is y:

$$y$$

- E is of the form $(F\ G)$:

$$(F[M/x]\ G[M/x])$$

- E is (lambda (x) E'):

$$(\text{lambda } (x)\ E')$$

- E is (lambda (y) E') and x is not free in E':

$$(\text{lambda } (y)\ E')$$

- E is (lambda (y) E') and y is not free in M:

$$(\text{lambda } (y)\ E'[M/x])$$

- E is (lambda (y) E'):

$$(\text{lambda } (z)\ E'[z/y, M/x])$$

  where z is not free in E' or M.