CS 530: Geometric and Probabilistic Methods in Computer Science
Homework 6 (Fall '06)

1. The n-th moment of Ψ is defined to be $M_n\{\Psi\} = \int_{-\infty}^{\infty} t^n \Psi(t) dt$. Let $f(t) = e^{-\pi t^2}$, $f'(t) = -2\pi t e^{-\pi t^2}$, and $f''(t) = 2\pi e^{-\pi t^2}(2\pi t^2 - 1)$. Prove the following:

   (a) $M_0\{f'\} = 0$.

   (b) $M_0\{f''\} = M_1\{f''\} = 0$.

2. The six vectors, $\mathbf{f}_1 = \begin{bmatrix} \cos(\pi/3) & \sin(\pi/3) \end{bmatrix}^{\mathrm{T}}$, $\mathbf{f}_2 = \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) \end{bmatrix}^{\mathrm{T}}$, $\mathbf{f}_3 = \begin{bmatrix} -1 & 0 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{f}_4 = \begin{bmatrix} -\cos(\pi/3) & -\sin(\pi/3) \end{bmatrix}^{\mathrm{T}}$, $\mathbf{f}_5 = \begin{bmatrix} -\cos(\pi/3) & \sin(\pi/3) \end{bmatrix}^{\mathrm{T}}$, and $\mathbf{f}_6 = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}}$ form a frame $\mathcal{F}$ for $\mathbb{R}^2$. Draw the frame.

   (a) Give two representations for the vector, $\mathbf{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}}$, in $\mathcal{F}$

   (b) Prove that $\mathbf{x}$ has an infinite number of representations in $\mathcal{F}$.

   (c) Give a matrix which transforms any representation of a vector in $\mathcal{F}$ into its representation in the standard basis for $\mathbb{R}^2$.

   (d) Give a matrix which transforms a representation of any vector in the standard basis for $\mathbb{R}^2$ into its representation in $\mathcal{F}$.

3. Let H_0 be the transfer function for the ideal lower halfband filter:

$$H_0(m) = \begin{cases} 0 \text{ if } \frac{M}{4} \leq m \leq \frac{3M}{4} - 1, \\ \sqrt{2} \text{ otherwise.} \end{cases}$$

   and let H_1 be the transfer function for the ideal upper halfband filter:

$$H_1(m) = \begin{cases} \sqrt{2} \text{ if } \frac{M}{4} \leq m \leq \frac{3M}{4} - 1, \\ 0 \text{ otherwise.} \end{cases}$$

   Use MATLAB to simulate a two channel subband coding system based on the ideal lower and upper halfband filters defined above. Test your subband coding system on row 215 of the Mars image found on the class homepage. Hand in listings of all MATLAB code and plots of the following:

(a) H_0 and H_1.

(b) $h_0 = \mathcal{F}^{-1}\{H_0\}$ and $h_1 = \mathcal{F}^{-1}\{H_1\}$.

(c) The time domain signal at the points labeled A-J in Figure 1.

(d) The frequency domain signal at the points labeled A-J in Figure 1.

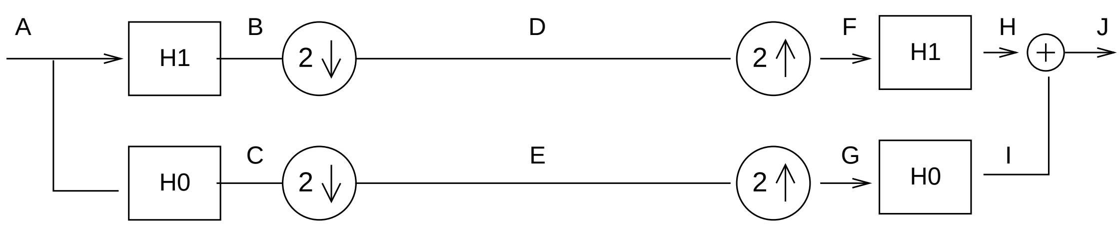

Figure 1: Two channel subband coding.